Math 1231 Fall 2025
Single-Variable Calculus I Section 11
Mastery Quiz 2
Due Monday, September 8

This week's mastery quiz has two topics. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

If you have a 2/2 on S1 (please check Blackboard Sunday night), you don't need to submit it again. If you have a 1/2 or 0/2 (or don't see a score), you need to submit it again.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 1: Estimation

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) Compute $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x^2 - 5x + 6}$.

Solution:

$$\begin{aligned}\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 5x + 6} &= \lim_{x\to 2} \frac{(x-2)(x+1)}{(x-2)(x-3)} \\ &= \lim_{x\to 2} \frac{x+1}{x-3} = \frac{3}{-1} = -3.\end{aligned}$$

(b) $\lim_{x\to 2} \dfrac{1}{x-2} - \dfrac{1}{x^2 - 3x + 2} =$

Solution:

$$\begin{aligned}\lim_{x\to 2} \frac{1}{x-2} - \frac{1}{x^2 - 3x + 2} &= \lim_{x\to 2} \frac{x-1}{(x-1)(x-2)} - \frac{1}{(x-1)(x-2)} \\ &= \lim_{x\to 2} \frac{x-2}{(x-1)(x-2)} = \lim_{x\to 2} \frac{1}{x-1} = 1.\end{aligned}$$

(c) Compute $\lim_{x\to 2} \dfrac{x^2 + x - 5}{3 - x} =$

Solution:

$$\lim_{x\to 2} \frac{x^2 + x - 5}{3 - x} = \frac{1}{1} = 1.$$

Secondary Topic 1: Estimation

(a) We want to amplify an electrical signal. Our amplifier will multiply the voltage by a factor of four, and we want an output signal of $20 \pm \varepsilon$ volts. Find a formula for δ in terms of ε, so that if the input error is less than δ then the error in the output is less than ε. Make sure your formula gives the **largest δ possible**, and justify your answer.

Solution: Our output error is $|4x - 20| = 4|x - 5|$, and we want this to be less than ε. So we get

$$|4x - 20| = 4|x - 5| < \varepsilon$$
$$|x - 5| < \varepsilon/4.$$

So if we take $\delta = \varepsilon/4$, then whenever the error in our input volahe $|x-5| < \delta$ then the error in our output voltage should be less than ε.

(b) Suppose $f(x) = 2x^2 + 2$, and we want an output of approximately 10. If we want our input to be positive, what input a should we aim for? Find a δ so that if our input is $a \pm \delta$ then our output will be 10 ± 1. Explain how you found this δ and why it should give us what we want.

Solution: We want an input of about $a = 2$. We want

$$\begin{aligned} |2x^2 + 2 - 10| &< 1 \\ |2x^2 - 8| &< 1 \\ 2|x-2||x+2| &< 1. \end{aligned}$$

Since $x \approx 2$ we can assume $|x+2| \approx 4$ and in particular $|x+2| < 5$. Then we want

$$\begin{aligned} 2|x-2||x+2| < 2|x-2| \cdot 5 &< 1 \\ |x-4| &< 1/10 \end{aligned}$$

So we take $\delta = 1/10$.