Math 1231 Fall 2025
Single-Variable Calculus I Section 12
Mastery Quiz 2
Due Wednesday, September 10

This week's mastery quiz has two topics. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

If you have a 2/2 on S1 (please check Blackboard Sunday night), you don't need to submit it again. If you have a 1/2 or 0/2 (or don't see a score), you need to submit it again. Everyone should submit M1.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 1: Estimation

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim_{x\to 3} \frac{1}{x-3} - \frac{3}{x^2-3x} =$

Solution:

$$\begin{aligned}\lim_{x\to 3} \frac{1}{x-3} + \frac{3}{x^2-3x} &= \lim_{x\to 3} \frac{x^2-3x-3(x-3)}{(x-3)(x^2-3x)} \\ &= \lim_{x\to 3} \frac{x^2-6x+9}{x(x-3)^2} \\ &= \lim_{x\to 3} \frac{1}{x} = \frac{1}{3}.\end{aligned}$$

(b) Compute $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} =$

Solution:

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} 1\sqrt{x+1}+2 = \frac{1}{4}.$$

(c) Compute $\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} =$

Solution:

$$\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} = \lim_{x\to 1} \frac{x^2-x-(x-1)}{(x-1)(x^2-x)} = \lim_{x\to 1} \frac{(x-1)^2}{x(x-1)^2} = \lim_{x\to 1} \frac{1}{x} = 1.$$

Secondary Topic 1: Estimation

(a) We want to build a ramp that's eight times long as it is tall, and we want it to reach a height of 10 meters. Find a formula for δ in terms of ϵ, so that if the error in the *length* is less than δ then the error in the height is less than ϵ. Make sure your formula gives the **largest** δ **possible**, and justify your answer.

Solution: The height is $L/8$, so our output error is $|L/8-10| = \frac{1}{8}|L-80|$, which we want to be less than ε. So we get

$$\begin{aligned}|L/8-10| = \frac{1}{8}|L-80| &< \varepsilon \\ |L-80| &< 8\varepsilon.\end{aligned}$$

So if we take $\delta = 8\varepsilon$, then whenever the error in the length of our ramp $|L - 80|$ is less than δ, then the error in our height should be less than ε.

(b) Suppose $f(x) = x^2 + 3x$, and we want an output of approximately 10. What input a should we aim for, if we want a positive input? Find a δ so that if our input is $a \pm \delta$ then our output will be 10 ± 1. Justify your answer.

Solution: We want an input of about $a = 2$. Our output error will be $|x^2+3x-10| = |x+5| \cdot |x-2|$. We know that $x + 5 \approx 7 < 8$, so we have

$$|x^2 + 3x - 10| = |x + 5| \cdot |x - 2| < 8|x - 2| < 1$$

so we need $|x - 2| < 1/8$. So we can take $\delta = 1/8$.