Math 1231 Fall 2025
Single-Variable Calculus I Section 12
Mastery Quiz 3
Due Wednesday, September 17

This week's mastery quiz has two topics. Everyone should submit both. (Even if you got a 2 on last week's M1, your grade depends on your best two attempts.) Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 2: Definition of Derivative

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim\limits_{x\to 1} \dfrac{\sin(3x-3)\sin(x-1)}{(x-1)^2} =$

Solution:

$$\begin{aligned}\lim_{x\to 1} \frac{\sin(3x-3)\sin(x-1)}{(x-1)^2} &= \lim_{x\to 1} \frac{\sin(3x-3)}{x-1}\frac{\sin(x-1)}{x-1} \\ &= \lim_{x\to 1} 3\frac{\sin(3x-3)}{3x-3} = 3.\end{aligned}$$

(b) $\lim\limits_{x\to -2} \dfrac{x-2}{(x+2)^2} =$

Solution: The top approaches -4 and the bottom approaches 0, so

$$\lim_{x\to -2} \frac{x-2}{(x+2)^2} = \pm\infty.$$

Further, we see that the top is negative and the bottom is always positive, so in fact the limit is $-\infty$.

(c) $\lim\limits_{x\to +\infty} \dfrac{\sqrt{3x^5+2x}}{x^{5/2}-x^{3/2}+1} =$

Solution:

$$\lim_{x\to +\infty} \frac{\sqrt{3x^5+2x}}{x^{5/2}-x^{3/2}+1} = \lim_{x\to +\infty} \frac{\sqrt{3+2/x^4}}{1-1/x+1/x^{5/2}} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

Secondary Topic 2: Definition of Derivative

(a) If $f(x) = x^2 + 2x$, find $f'(2)$, explicitly using the definition of the derivative.

Solution:

$$\begin{aligned}f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \frac{(2+h)^2+2(2+h)-8}{h} \\ &= \lim_{h\to 0} \frac{4+4h+h^2+4+2h-8}{h} \\ &= \lim_{h\to 0} \frac{h^2+6h}{h} \\ &= \lim_{h\to 0} h+6 = 6.\end{aligned}$$

(b) If $g(x) = \sqrt{x-5}$, find $g'(x)$, directly from the formal definition of the derivative.

Solution:

$$\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h-5}-\sqrt{x-5}}{h} \\
&= \lim_{h\to 0} \frac{x+h-5-(x-5)}{h(\sqrt{x+h-5}+\sqrt{x-5})} \\
&= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h-5}+\sqrt{x-5})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h-5}+\sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}.
\end{aligned}$$