Math 1231: Single-Variable Calculus 1
George Washington University Fall 2025
Recitation 3

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Problem 1. Let $h(x) = \frac{x-1}{\sqrt{5-x}-2}$.

(a) Is this function continuous where it's defined? Where is it undefined?

(b) We can factor an $x-1$ out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the $x-1$ appear. What tricks do we have that might work?

(c) What is $\lim_{x\to 1} h(x)$?

Problem 2. We want to compute $\lim_{x\to 3} \frac{\sin(x^2-9)}{x-3}$.

(a) What rule do we know we need to invoke here?

(b) What θ are we going to need to pick for this to work out, and why?

(c) Do algebra so that you can invoke the small angle approximation. What is the limit? (Are you using the AIF property?)

(d) Go back to the beginning, and see what our heuristic idea that $\sin(\theta) \approx \theta$ would have told you. Does that match with what you got?

Problem 3. We want to think about the ways that infinity doesn't really work like a number, and we can't do arithmetic with it.

(a) To start: what is $\lim_{x\to 0} 1/x$, and why?

(b) Let's look at $\lim_{x\to 0} 1/x + 1/x$. If we computed the limit of each fraction individually, what indeterminate form would we get?

(c) How do we actually compute $\lim_{x\to 0} \frac{1}{x} + \frac{1}{x}$? (Hint: combine them into one fraction.) Does this make sense in light of what you got in part (b)?

(d) Now consider $\lim_{x\to 0} \frac{1}{x} + \frac{x-1}{x-x^2}$. What is the limit of each piece, and what indeterminate form is this?

(e) Compute $\lim_{x\to 0} \frac{1}{x} + \frac{x-1}{x-x^2}$ directly. Does this make sense in light of what you got in part (d)?

(f) Now consider $\lim_{x\to 0} 1/x + 1/x^2$. What indeterminate form would this represent? What is the limit? Do those make sense together?

(g) Finally, let's look at $\lim_{x\to 0} \frac{1}{x} + \frac{x^2-3x+2}{x^2-2x}$. What indeterminate form is this? What is the limit?

(h) What pattern do you see from all of these?