Math 1231: Single-Variable Calculus 1
George Washington University Fall 2025
Recitation 3

Jay Daigle

September 9, 2025

We want to practice the way we actually compute limits. Remember we have two key principles:

(a) Functions built out of algebra and trigonometry are *continuous* wherever they're defined. When a function is continuous, we can compute the limit by just plugging in.

(b) We say two functions are *almost identical* if they're the same except at a small collection of points. If two functions are almost identical, then their limits will always be the same. This allows us to replace complicated functions with simpler ones.

Problem 1 (Warmup)**.** Let $f(x) = \frac{x^2+\sin(x)+3}{x^2-x-2}$.

(a) Where is f continuous? Where is it discontinuous?

(b) What is $\lim_{x\to 0} f(x)$?

Problem 2. Let $f(x) = \frac{x-1}{x^2-1}$.

(a) What is $f(2)$? Is f continuous at 2?

(b) What is $\lim_{x\to 2} f(x)$?

(c) What is $f(1)$? Is f continuous at 1?

(d) What function can we find that's almost the same as f, but defined and continuous at 1? (Is this function the same as f?)

(e) What is $\lim_{x\to 1} f(x)$?

Problem 3. Let $g(x) = \frac{(x+1)^2-1}{x+2}$.

(a) Is g continuous where it's defined? Where is it undefined?

(b) Can you find a function that's almost identical to g but continuous everywhere?

(c) What is $\lim_{x\to -2} g(x)$?

Problem 4. Let $h(x) = \frac{x-1}{\sqrt{5-x}-2}$.

(a) Is this function continuous where it's defined? Where is it undefined?

(b) We can factor an $x-1$ out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the $x-1$ appear. What tricks do we have that might work?

(c) What is $\lim_{X\to 1} h(x)$?

Problem 5. We want to compute $\lim_{x\to 3} \frac{\sin(x^2-9)}{x-3}$.

(a) What rule do we know we need to invoke here?

(b) What θ are we going to need to pick for this to work out, and why?

(c) Do algebra so that you can invoke the small angle approximation. What is the limit? (Are you using the AIF property?)

(d) Go back to the beginning, and see what our heuristic idea that $\sin(\theta) \approx \theta$ would have told you. Does that match with what you got?