

Math 1231 Fall 2025

Single-Variable Calculus I Section 11

Mastery Quiz 4

Due Monday, September 22

This week's mastery quiz has three topics. Everyone should submit M2 and S2. If you have a 4/4 on Blackboard in M1, you don't need to submit it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Major Topic 2: Computing Derivatives
- Secondary Topic 2: Definition of Derivative

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim_{x \rightarrow +\infty} \frac{3x^2 + 2x + 1}{\sqrt{x^4 - x^2 + x}} =$

Solution:

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{3x^2 + 2x + 1}{\sqrt{x^4 - x^2 + x}} &= \lim_{x \rightarrow +\infty} \frac{3 + 2/x + 1/x^2}{1/\sqrt{x^4} \sqrt{x^4 - x^2 + x}} \\ &= \lim_{x \rightarrow +\infty} \frac{3 + 2/x + 1/x^2}{\sqrt{1 - 1/x^2 + 1/x^3}} \\ &= \frac{3 + 0 + 0}{\sqrt{1 - 0 + 0}} = 3. \end{aligned}$$

(b) $\lim_{x \rightarrow -2} \frac{x + 2}{\sqrt{x + 6} - 2} =$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -2} \frac{x + 2}{\sqrt{x + 6} - 2} &= \lim_{x \rightarrow -2} \frac{(x + 2)(\sqrt{x + 6} + 2)}{x + 6 - 4} \\ &= \lim_{x \rightarrow -2} \frac{\sqrt{x + 6} + 2}{1} = 4. \end{aligned}$$

(c) $\lim_{x \rightarrow 0} \frac{\sin(5x^2) + \tan^2(x)}{x^2} =$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(5x^2) + \tan^2(x)}{x^2} &= \lim_{x \rightarrow 0} 5 \frac{\sin(5x^2)}{5x^2} + \lim_{x \rightarrow 0} \left(\frac{\sin(x)}{\cos(x) \cdot x} \right)^2 \\ &= 5 + 1^2 = 6 \end{aligned}$$

by the Small Angle Approximation.

Major Topic 2: Computing Derivatives

- (a) Compute the derivative of $f(x) = (2x^4 + 3)(5x - 2\sqrt{x})$, explicitly justifying each step and naming each derivative rule you use.

Solution:

$$\begin{aligned}
f'(x) &= (2x^4 + 3)'(5x - 2\sqrt{x}) + (2x^4 + 3)(5x - 2\sqrt{x})' && \text{Product Rule} \\
&= \left((2x^4)' + (3)'\right)(5x - 2\sqrt{x}) + (2x^4 + 3)\left((5x)' - (2\sqrt{x})'\right) && \text{Sum Rule} \\
&= \left((2x^4)' + 0\right)(5x - 2\sqrt{x}) + (2x^4 + 3)\left((5x)' - (2\sqrt{x})'\right) && \text{Constants Rule} \\
&= \left(2(x^4)'\right)(5x - 2\sqrt{x}) + (2x^4 + 3)\left(5(x)' - 2(\sqrt{x})'\right) && \text{Scalar Products} \\
&= \left(2(x^4)'\right)(5x - 2\sqrt{x}) + (2x^4 + 3)\left(5 \cdot 1 - 2(\sqrt{x})'\right) && \text{Identity} \\
&= (2 \cdot 4x^3)(5x - 2\sqrt{x}) + (2x^4 + 3)\left(5 \cdot 1 - 2 \cdot \frac{1}{2}x^{-1/2}\right) && \text{Power Rule.}
\end{aligned}$$

(b) Compute the derivative of $g(x) = \frac{5x^4 - 3x^2}{x^5 + \sqrt[5]{x} + 7}$.

Solution:

$$g'(x) = \frac{(20x^3 - 6x)(x^5 + \sqrt[5]{x} + 7) - (5x^4 + \frac{1}{5}x^{-4/5})(5x^4 - 3x^2)}{(x^5 + x + 7)^2}.$$

Secondary Topic 2: Definition of Derivative

(a) If $f(x) = \sqrt{x+3}$, find $f'(6)$, directly from the definition of derivative.

Solution:

$$\begin{aligned}
f'(2) &= \lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{9+h} - 3}{h} \\
&= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{9+h} + 3)} \\
&= \lim_{h \rightarrow 0} \frac{1}{\sqrt{9+h} + 3} \\
&= \lim_{h \rightarrow 0} \frac{1}{6}.
\end{aligned}$$

(b) If $g(x) = \frac{1}{x+2}$, find $g'(a)$, explicitly using the definition of the derivative.

Solution:

$$\begin{aligned}g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+2) - (x+h+2)}{(x+2)(x+h+2)h} \\&= \lim_{h \rightarrow 0} \frac{-h}{(x+2)(x+h+2)h} \\&= \lim_{h \rightarrow 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2}.\end{aligned}$$