Math 1231 Fall 2025
Single-Variable Calculus I Section 12
Mastery Quiz 4
Due Wednesday, September 24

This week's mastery quiz has three topics. Everyone should submit topic M2. If you have a 2/2 on S2 you don't need to submit it this week; if you have a 4/4 on M1 you don't need to submit it this week. (Check Blackboard for your current scores!)

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Major Topic 2: Computing Derivatives
- Secondary Topic 2: Definition of Derivative

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim\limits_{x\to-2} \dfrac{\tan(2x+4)}{x+2}$

Solution:

$$\lim_{x\to-2} \frac{\tan(2x+4)}{x+2} = \lim_{x\to-2} \frac{\sin(2x+4))\cdot 2}{(2x+4)\cdot\cos(2x+4)} = 1\cdot\frac{2}{1} = 2.$$

(b) $\lim\limits_{x\to1} \dfrac{\sqrt{x+1}-2}{x-1}$

Solution: The limit of the top is $\sqrt{2}-2$ and the limit of the bottom is zero, so the limit is $\pm\infty$. Since the bottom can be positive or negative, we can't be any more specific.

We could, if we wanted, multiply by the conjugate. That would give us

$$\lim_{x\to1} \frac{\sqrt{x+1}-2}{x-1} = \lim_{x\to1} \frac{x-3}{(x-1)(\sqrt{x+1}+2)}$$

and now the numerator goes to -2 and the bottom goes to zero. So again we see the limit is $\pm\infty$.

(c) Compute $\lim\limits_{x\to3} \dfrac{x^2-5x+6}{x^2-x-6} =$

Solution:

$$\begin{aligned}\lim_{x\to3} \frac{x^2-5x+6}{x^2-x-6} &= \lim_{x\to3} \frac{(x-3)(x-2)}{(x-3)(x+2)} \\ &= \lim_{x\to3} \frac{x-2}{x+2} = 1/5.\end{aligned}$$

Major Topic 2: Computing Derivatives

(a) Explicitly justifying each step and naming each derivative rule you use, compute $\frac{d}{dx}\frac{\sin(x)+1}{2x^2-5}$.

Solution:

$$\begin{aligned}
\frac{d}{dx}\frac{\sin(x)+1}{2x^2-5} &= \frac{(\sin(x)+1)'(2x^2-5)-(2x^2-5)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Quotient rule}\\
&= \frac{\big((\sin(x))'+(1)'\big)(2x^2-5)-\big((2x^2)'-(5)'\big)(\sin(x)+1)}{(2x^2-5)^2} && \text{Sum Rule}\\
&= \frac{(\sin(x))'(2x^2-5)-(2x^2)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Constants rule}\\
&= \frac{\cos(x)(2x^2-5)-(2x^2)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Derivative of sine}\\
&= \frac{\cos(x)(2x^2-5)-2(x^2)'(\sin(x)+1)}{(2x^2-5)^2} && \text{Scalar Products}\\
&= \frac{\cos(x)(2x^2-5)-2(2x)(\sin(x)+1)}{(2x^2-5)^2} && \text{Power Rule.}
\end{aligned}$$

(b) Compute the derivative of $h(x) = \frac{5}{x^4}$.

Solution: There are two ways you could approach this but you need to do it correctly consistently; this is question that trips people up very frequently.

One option is to use the quotient rule. But if we do that we have to remember that $5' = 0$. So we get

$$h'(x) = \frac{(5)'x^4 - 5(x^4)'}{(x^4)^2} = \frac{0-20x^3}{x^8} = \frac{-20}{x^5}.$$

The other option is to rewrite this as an exponent. Then we have

$$h'(x) = \frac{d}{dx}5x^{-4} = 5\cdot(-4x^{-5}) = -20x^{-5}.$$

I think the second approach is better and more consistent. But you *can* use the quotient rule as long as you do it correctly.

(c) Compute $\frac{d}{dx}(5x^7-3x)\left(x^{4/3}+\frac{1}{x}\right)$

Solution:

$$(35x^6-3)\left(x^{4/3}+\frac{1}{x}\right)+\left(\frac{4}{3}x^{1/3}-\frac{1}{x^2}\right)(5x^7-3x).$$

Secondary Topic 2: Definition of Derivative

(a) **Directly from the definition of derivative,** compute the derivative of $f(x) = x^2+\sqrt{x}$ at $a = 2$.

Solution:

$$
\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\
&= \lim_{h\to 0} \frac{(2+h)^2+\sqrt{2+h}-2^2-\sqrt{2}}{h} \\
&= \left(\lim_{h\to 0} \frac{4h+h^2}{h}\right) + \left(\lim_{h\to 0} \frac{(\sqrt{2+h}-\sqrt{2})(\sqrt{2+h}+\sqrt{2})}{h(\sqrt{2+h}+\sqrt{2})}\right) \\
&= \left(\lim_{h\to 0} 4+h\right) + \left(\lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}\right) \\
&= 4 + \frac{1}{2\sqrt{2}}.
\end{aligned}
$$

(b) If $g(x) = \frac{x}{x+2}$, find $g'(x)$ explicitly using the definition of derivative.

Solution:

$$
\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{x+h}{x+h+2}-\frac{x}{x+2}}{h} \\
&= \lim_{h\to 0} \frac{(x+h)(x+2)-x(x+h+2)}{h(x+2)(x+h+2)} \\
&= \lim_{h\to 0} \frac{x^2+hx+2x+2h-x^2-xh-2x}{h(x+2)(x+h+2)} \\
&= \lim_{h\to 0} \frac{2h}{h(x+2)(x+h+2)} \\
&= \lim_{h\to 0} \frac{2}{(x+2)(x+h+2)} = \frac{2}{(x+2)^2}.
\end{aligned}
$$