Math 1231: Single-Variable Calculus 1

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Recitation 4

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Problem 1. Let $f(x) = x^3$. We want to find a formula for the derivative of this function at any given point.

(a) Write down a formula for $f'(a)$ using the $h \to 0$ limit formulation. What does the numerator mean? What does the denominator mean?

(b) Use your formula from part (a) to compute the derivative.

(c) Now write down a formula for $f'(a)$ using the $x \to a$ limit formulation. Does this look easier or harder than the formula from part (a), and why? What does the numerator mean? What does the denominator mean?

(d) Use the formula from part (c) to compute the derivative. You should get the same answer you got in part (b).

(e) Which method was faster? Which method was easier?

Problem 2. Let $g(x) = \frac{1}{x+3}$.

(a) Write down a limit expression to compute $g'(2)$. Be careful with order of operations and parentheses!

(b) Now compute $g'(2)$.

(c) Write a limit expression to compute $g'(x)$. Again, make sure you get your order of operations right.

(d) Compute $g'(x)$.

Problem 3. Let $a(x) = |x|$ be the absolute value function.

(a) Write down a formula for a as a piecewise function.

(b) Write down a limit expression for the derivative of a at 0.

(c) What is the limit from the right?

(d) What is the limit from the left?

(e) What does that tell you about the derivative?

Problem 4 (Bonus)**.** Let $g(x) = \sqrt[3]{x}$.

(a) Write down a limit formula to compute the derivative of g at 0.

(b) What is $g'(0)$? What does this tell you?

(c) Now write down a limit formula to compute the derivative of $p(x) = \sqrt[3]{x^2}$.

(d) What is this limit? What does that tell you?

(e) Write down a limit formula to compute the derivative of g at a when $a \neq 0$.

(f) Can you compute this limit? What do you have to do here? (It's not obvious, but there's an algebraic trick from Day 1 that can help us.)

Problem 5 (Bonus)**.** Let $f(x) = \sqrt{x^2 - 4}$.

(a) Set up a limit expression to calculate $f'(x)$. Do you think $h \to 0$ or $x \to a$ will be easier here?

(b) Compute $f'(x)$.

(c) Where is f differentiable? Where is it not differentiable?

We can use the limit definition of the derivative to compute derivatives, but it's not very efficient. In class we learned some rules for computing them more easily:

- (Constants) $\frac{d}{dx}c = 0$
- (Identity) $\frac{d}{dx}x = 1$

- (Scalar products) $\frac{d}{dx}cf(x) = cf'(x)$
- (Sum Rule) $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
- (Power Rule) $\frac{d}{dx}x^n = nx^{n-1}$
- (Product Rule) $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
- (Quotient Rule) $\dfrac{d}{dx}\dfrac{f(x)}{g(x)} = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$.

Problem 6. Let's find the derivative of $3x^2 + 5x$.

(a) First just plug 1 into this function. Think about what operations you do in which order. Which operation do you do last?

(b) What rule should we invoke first? Use that rule to write this as a combination of two smaller derivative problems.

(c) What rule do we use next? We need to use it in two places.

(d) Finish off computing this derivative. What rules did you use?

Problem 7. (a) Use the product rule to differentiate $(x^2 + 1)(3x^3 - 5)$.

(b) Multiply out $(x^2 + 1)(3x^3 - 5)$ to get one big polynomial. Use our derivative rules to compute that derivative.

(c) Which process was easier?

Problem 8. Compute $\dfrac{d}{dx}\dfrac{x^5 - 7x}{4x^2 + 3}$.