

Math 1231: Single-Variable Calculus 1
George Washington University Fall 2025
Recitation 4

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Problem 1. We want to think about the ways that infinity doesn't really work like a number, and we can't do arithmetic with it.

- (a) To start: what is $\lim_{x \rightarrow 0} 1/x$, and why?
- (b) Let's look at $\lim_{x \rightarrow 0} 1/x + 1/x$. If we computed the limit of each fraction individually, what indeterminate form would we get?
- (c) How do we actually compute $\lim_{x \rightarrow 0} \frac{1}{x} + \frac{1}{x}$? (Hint: combine them into one fraction.) Does this make sense in light of what you got in part (b)?
- (d) Now consider $\lim_{x \rightarrow 0} \frac{1}{x} + \frac{x-1}{x-x^2}$. What is the limit of each piece, and what indeterminate form is this?
- (e) Compute $\lim_{x \rightarrow 0} \frac{1}{x} + \frac{x-1}{x-x^2}$ directly. Does this make sense in light of what you got in part (d)?
- (f) Now consider $\lim_{x \rightarrow 0} 1/x + 1/x^2$. What indeterminate form would this represent? What is the limit? Do those make sense together?
- (g) Finally, let's look at $\lim_{x \rightarrow 0} \frac{1}{x} + \frac{x^2-3x+2}{x^2-2x}$. What indeterminate form is this? What is the limit?
- (h) What pattern do you see from all of these?

Problem 2. (a) Consider $\lim_{x \rightarrow -\infty} \frac{x}{x+1}$. Can you come up with a heuristic guess about what this limit is?

- (b) Can you carefully justify your guess from part (a).
- (c) Now consider $\lim_{x \rightarrow +\infty} \frac{x}{\sqrt{3x^2+x}}$, and come up with a heuristic estimate for the limit.
- (d) Carefully justify your guess from part (c).
- (e) How would either of those calculations change if we take the limit to the other infinity?

Problem 3.

- (a) We want to compute $\lim_{x \rightarrow +\infty} \sqrt{x^2+1} - x$. Can we just plug in here, or is this an indeterminate form? Why?
- (b) When we have an indeterminate form, we generally want to write it as a big fraction, simplify, and factor. How can we do that here? We have to use a technique from last week to really get this to work.
- (c) Once you have a big fraction, use it to compute the limit.
- (d) How does this argument change if instead we want $\lim_{x \rightarrow +\infty} \sqrt{x^2+x+1} - x$?
- (e) What is $\lim_{x \rightarrow +\infty} \sqrt{x^2+ax+1} - x$?
- (f) What does the answer in part (e) say about $\lim_{x \rightarrow +\infty} \sqrt{x^2+2x+1} - x$? Why should the answer to this question be obvious?

Problem 4. Let $f(x) = x^3$. We want to find a formula for the derivative of this function at any given point.

- (a) Write down a formula for $f'(a)$ using the $h \rightarrow 0$ limit formulation. What does the numerator mean? What does the denominator mean?
- (b) Use your formula from part (a) to compute the derivative.
- (c) Now write down a formula for $f'(a)$ using the $x \rightarrow a$ limit formulation. Does this look easier or harder than the formula from part (a), and why? What does the numerator mean? What does the denominator mean?
- (d) Use the formula from part (c) to compute the derivative. You should get the same answer you got in part (b).
- (e) Which method was faster? Which method was easier?