Math 1231 Fall 2025
Single-Variable Calculus I Section 11
Mastery Quiz 5
Due Monday, September 29

This week's mastery quiz has four topics. Everyone should submit M2 and S3. If you have a 4/4 on Blackboard in M1, you don't need to submit it again. If you have a 2/2 on S2, you don't need to submit it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Major Topic 2: Computing Derivatives
- Secondary Topic 2: Definition of Derivative
- Secondary Topic 3: Linear Approximation

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim_{x \to 1} \frac{\sqrt{x+3}-2}{x-1} =$

Solution:

$$\begin{aligned}\lim_{x \to 1} \frac{\sqrt{x+3}-2}{x-1} &= \lim_{x \to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} \\ &= \lim_{x \to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}.\end{aligned}$$

(b) $\lim_{x \to -1} \frac{1-x}{1+x} =$

Solution: The limit of the top is 2 and the limit of the bottom is 0, so the limit is $\pm\infty$. Since the denominator can be positive or negative, we can't be more specific.

(c) Compute $\lim_{x \to 0} \frac{\sin(3x)\sin(4x)}{x\sin(2x)} =$

Solution:

$$\begin{aligned}\lim_{x \to 0} \frac{\sin(3x)\sin(4x)}{x\sin(2x)} &= \lim_{x \to 0} \frac{\sin(3x)}{3x} \frac{\sin(4x)}{4x} \frac{12x^2}{x\sin(2x)} \\ &= \lim_{x \to 0} \frac{2x}{\sin(2x)} \frac{6x}{x} \\ &= \lim_{x \to 0} \frac{6x}{x} = 6.\end{aligned}$$

Major Topic 2: Computing Derivatives

(a) Compute $\frac{d}{dx} \sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right) =$

Solution:

$$\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right) \tan\left(\frac{x^2+1}{\sqrt{x^3-2}}\right) \frac{2x\sqrt{x^3-2} - (x^2+1)\frac{1}{2}(x^3-2)^{-1/2} \cdot 3x^2}{x^3-2}.$$

(b) Compute $\frac{d}{dx} \cos^2\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)$.

Solution:

$$\begin{aligned} &2\cos\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)\left(-\sin\left(\tan^2\left(\sec^2(\sqrt{x}+x)\right)\right)\right) \\ &\cdot 2\tan\left(\sec^2(\sqrt{x}+x)\right)\sec^2\left(\sec^2(\sqrt{x}+x)\right) \\ &\cdot 2\sec(\sqrt{x}+x)\sec(\sqrt{x}+x)\tan(\sqrt{x}+x)\left(\frac{1}{2x}+1\right) \end{aligned}$$

Secondary Topic 2: Definition of Derivative

(a) If $f(x) = \frac{x+1}{x-1}$, find $f'(2)$ directly from the formal definition of the derivative.

Solution:

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \\ &= \lim_{h\to 0} \frac{\frac{3+h}{1+h}-3}{h} \\ &= \lim_{h\to 0} \frac{1}{h}\frac{3+h-3(1+h)}{1+h} \\ &= \lim_{h\to 0} \frac{-2h}{h(1+h)} \\ &= \lim_{h\to 0} \frac{-2}{1+h} = -2. \end{aligned}$$

(b) If $g(x) = x^3 - 3x$, find $g'(x)$, directly from the definition of derivative.

Solution:

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^3-3(x+h)-x^3+3x}{h} \\ &= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-3x-3h-x^3+3x}{h} \\ &= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-3h}{h} \\ &= \lim_{h\to 0} 3x^2+3xh+h^2-3 = 3x^2-3. \end{aligned}$$

Secondary Topic 3: Linear Approximation

(a) Give a formula for a linear approximation of $f(x) = x\sqrt{x+1}$ near the point $a = 3$.

Use your answer to estimate $f(2.8)$.

Solution:

$$\begin{aligned} f'(x) &= \sqrt{x+1} + \frac{x}{2\sqrt{x+1}} \\ f'(3) &= 2 + \frac{3}{4} = \frac{11}{4} \\ f(x) &\approx f(a) + f'(a)(x-a) = 6 + \frac{11}{4}(x-3). \end{aligned}$$

$f(2.8) \approx 6 + \frac{11}{4}(-.2) = 6 - \frac{11}{20} = \frac{109}{20}$.

(b) Give a formula for a linear approximation to $g(x) = \sin(x^2 - 3x)$ near the point $a = 0$.

Solution:

$$\begin{aligned} g'(x) &= \cos(x^2 - x)(2x - 3) \\ g'(0) &= 1 \cdot (0 - 3) = -3 \\ g(x) &\approx 0 - 3(x - 0) \\ &= -3x. \end{aligned}$$