Math 1231 Fall 2025

Single-Variable Calculus I Section 12

Mastery Quiz 5

Due Wednesday, October 1

This week's mastery quiz has four topics. Everyone should submit topics M2, S3, and S4. If you have a 4/4 on M1 you don't need to submit it this week. (Check Blackboard for your current scores!) This quiz is the last quiz for M1.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Major Topic 2: Computing Derivatives
- Secondary Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim_{x \to 1} \dfrac{\sqrt{x+3}-2}{x-1} =$

Solution:

$$\begin{aligned}\lim_{x \to 1} \frac{\sqrt{x+3}-2}{x-1} &= \lim_{x \to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} \\ &= \lim_{x \to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}.\end{aligned}$$

(b) $\lim_{x \to +\infty} \dfrac{3x^2+2x+1}{\sqrt{x^4-x^2+x}} =$

Solution:

$$\begin{aligned}\lim_{x \to +\infty} \frac{3x^2+2x+1}{\sqrt{x^4-x^2+x}} &= \lim_{x \to +\infty} \frac{3+2/x+1/x^2}{1/\sqrt{x^4}\sqrt{x^4-x^2+x}} \\ &= \lim_{x \to +\infty} \frac{3+2/x+1/x^2}{\sqrt{1-1/x^2+1/x^3}} \\ &= \frac{3+0+0}{\sqrt{1-0+0}} = 3.\end{aligned}$$

(c) $\lim_{x \to 4^+} \dfrac{x+1}{x-4} =$

Solution: The limit of the top is 5 and the limit of the bottom is 0, so the limit is $\pm\infty$. Since the bottom will always be positive as we approach from the right, the overall limit is in fact $+\infty$.

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx} \tan^{3/5}\left(\sec\left(x^3-4\right)\right)$

Solution:

$$\begin{aligned}&\frac{3}{5}\tan^{-2/5}\left(\sec\left(x^3-4\right)\right)\sec^2\left(\sec\left(x^3-4\right)\right) \\ &\qquad \cdot \sec\left(x^3-4\right)\tan\left(x^3-4\right)3x^2\end{aligned}$$

(b) Compute $\dfrac{d}{dt}\sqrt[5]{\dfrac{\tan^2(t^2+1)+2}{\sin(2t)-2t}}$.

Solution:

$$\frac{d}{dt}\sqrt[5]{\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}} = \frac{1}{5}\left(\frac{\tan^2(t^2+1)+2}{\sin(2t)-2t}\right)^{-4/5} \cdot \frac{\left(2\tan(t^2+1)\sec^2(t^2+1)2t\right)\left(\sin(2t)-2t\right)-\left(2\cos(2t)-2\right)\left(\tan^2(t^2+1)+2\right)}{\left(\sin(2t)-2t\right)^2}.$$

Secondary Topic 3: Linear Approximation

(a) Estimate $\sqrt[4]{15}$ using a linear approximation of the function $\sqrt[4]{x}$ at the point 16.

Solution: We have $h(x) = \sqrt[4]{x}$ and so $h'(x) = \frac{1}{4}x^{-3/4} = \frac{1}{4\sqrt[4]{x^3}}$. Thus in particular, we have $h'(16) = \frac{1}{4\sqrt[4]{16^3}} = \frac{1}{4\cdot 2^3} = 1/32$.

The tangent line approximation is

$$y - 2 = \frac{1}{32}(x-16)$$

so we have

$$\begin{aligned} f(x) &\approx \frac{1}{32}(x-16)+2 \\ f(15) &\approx \frac{1}{32}(-1)+2 = 2-\frac{1}{32} = \frac{63}{32}. \end{aligned}$$

(b) Give a formula for a linear approximation to $g(x) = 2x - \tan(x)$ near the point $a = \pi$.

Solution:

$$\begin{aligned} g(\pi) &= 2\pi - 0 = \pi \\ g'(x) &= 2 - \sec^2(x) \\ g'(\pi) &= 2 - 1 = 1 \\ g(x) &\approx 2\pi + (x-\pi) \end{aligned}$$

Secondary Topic 4: Rates of Change

(a) Suppose that a factory produces widgets, and if p people work at the factory then they will produce a total of $W(p) = 30\sqrt{p}$ widgets.

(i) What are the units of $W'(p)$? What does it represent physically? What does it mean if W' is big?

(ii) Calculate $W'(9)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) $W'(p)$ has units of widgets per person. It describes the rate at which the number of widgets increases as we add more people to the factory (this is called the marginal product of labor, but you don't need to know that). If it's large, that means that adding one more person to the factory will let us produce a lot more widgets.

(ii) $W'(p) = \frac{15}{\sqrt{p}}$ so $W'(9) = 5$. So moving from nine people to ten people working at the factory will lead to the production of five extra widgets.

(b) Suppose the height of a particle in centimeters is given as a function of time in seconds $p(t) = t^3 - 3t$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $p'(t) = 3t^2 - 3$ is zero when $t = \pm 1$ second.

(ii) $p''(t) = 6t$ is zero when $t = 0$ seconds.