

Math 1231: Single-Variable Calculus 1
George Washington University Fall 2025
Recitation 5

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Problem 1. Consider the function $\sec^2(x^2 + 1)$

- (a) Find functions f and g so that $(f \circ g)(x) = \sec^2(x^2 + 1)$.
- (b) Talk to the people next to you. Did they pick the same f and g that you did? Can you find a different pair of functions f and g that also work?
- (c) Find functions f, g, h so that $(f \circ g \circ h)(x) = \sec^2(x^2 + 1)$.
- (d) Compute f', g' , and h' .
- (e) What is $\frac{d}{dx} \sec^2(x^2 + 1)$?

Problem 2. Find

$$\frac{d}{dx} \frac{\sin(x^2) + \sin^2(x)}{x^2 + 1}$$

Problem 3. (a) Compute

$$\frac{d}{dx} \sqrt{\frac{\sqrt{x} + 1}{(\cos x + 1)^2}}$$

(b) Find

$$\frac{d}{dx} \tan^4(\sqrt[3]{x^5 + x^3 + 2} + 1).$$

Problem 4 (Bonus). Calculate

$$\frac{d}{dx} \left(\frac{\sin^2\left(\frac{x^2+1}{\sqrt{x-1}}\right) + \sqrt{x^3-2}}{\cos(\sqrt{x^2+1}+1) - \tan(x^4+3)} \right)^{5/3}$$

Problem 5 (Geometric Series). One function it's sometimes important to approximate is the “geometric series” formula $f(x) = \frac{1}{1-x}$, near $x = 0$.

- (a) What is $f'(x)$?
- (b) Find a linear approximation for $f(x)$ near $x = 0$.
- (c) Use this formula to estimate $\frac{1}{.9}$ and $\frac{1}{1.01}$. Do these answers make sense?
- (d) Use your formula to estimate $\frac{1}{1.5}$ and $\frac{1}{10.5}$. Do these answers make sense?
- (e) Use your formula to estimate $f(-1)$ and $f(1)$. Do these answers make sense?

Problem 6. (a) Use the binomial approximation to estimate $\sqrt{2}$ and $\sqrt[3]{2}$.

- (b) Use the binomial approximation to estimate $\sqrt{17}$. (Remember: 17 is not close to 1! You need to be slightly clever here.)
- (c) Can you find a formula to approximate $(1+x^n)^\alpha$ for a real number α ?
- (d) What does this tell us about $\sqrt{1+x^2}$?

Problem 7 (Bonus). Find a formula to approximate $f(x) = x^3 + 3x^2 + 5x + 1$ near $a = 0$. What do you notice? Why does that happen?