Math 1231 Fall 2025
Single-Variable Calculus I Section 12
Mastery Quiz 6
Due Wednesday, October 8

This week's mastery quiz has three topics. If you have a 4/4 on M1 you don't need to submit it this week. If you have a 2/2 on S3 or S4, you don't need to submit it. (Check Blackboard for your current scores!) This quiz is the last quiz for S3 and S4.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 3: Linear Approximation
- Secondary Topic 4: Rates of Change

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx}\dfrac{\sin(\csc(x^2+1))}{x^4+\cos(x)} =$

Solution:

$$\frac{\big(\cos(\csc(x^2+1))(-\csc(x^2+1)\cot(x^2+1))2x\big)(x^4+\cos(x)) - (4x^3-\sin(x))\sin(\csc(x^2+1))}{(x^4+\cos(x))^2}.$$

(b) Compute $\dfrac{d}{dx}\tan(x^2+\csc^2(x^3+\sin(x^5)))$.

Solution:

$$\frac{d}{dx}\tan(x^2+\csc(x^3+\sin(x^5)))$$
$$= \sec^2(x^2+\csc(x^3+\sin(x^5)))\Big(2x-2\csc^2(x^3+\sin(x^5))\cot(x^3+\sin(x^5))\Big)\cdot(3x^2+\cos(x^5)\cdot 5x^4).$$

Secondary Topic 3: Linear Approximation

(a) Give a formula for a linear approximation of $f(x)=\sqrt{x^3+1}$ near the point $a=2$.

Use your answer to estimate $f(2.1)$.

Solution:

$$\begin{aligned} f'(x) &= \frac{1}{2}(x^3+1)^{-1/2}3x^2 \\ f'(a) &= \frac{1}{2}(9)^{-1/2}\cdot 12 = 2 \\ f(x) &= f(a)+f'(a)(x-a) = 3+2(x-2). \end{aligned}$$

$f(2.1)\approx 3+2(.1)=3.2.$

(b) Find an equation of the line tangent to $y=\frac{x+1}{x-1}$ at the point $x=2$.

Solution:

$$y' = \frac{(x-1)-(x+1)}{(x-1)^2}$$

and thus at $x=2$ we have $y'=\frac{1-3}{1^2}=-2$. The point on the curve is $(2,3)$, so we have the equation

$$y-3=-2(x-2) \qquad \text{or} \qquad y=7-2x.$$

Secondary Topic 4: Rates of Change

(a) Let $F(x) = \frac{1}{x} + 1$ be the amount of pressure exerted on a beam in pounds per square inch at a point x inches to the right of its left end.

(i) What are the units of $F'(x)$? What does $F'(x)$ represent physically? What would it mean if $F'(x)$ is big?

(ii) Compute $F'(5)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) The derivative $F'(x)$ has units pounds per square inch per inch, or pounds per cubic inch. $F'(x)$ is the rate at which pressure is increasing as you move to the right along the stick. If $F'(x)$ is big, that means that moving along the stick a little bit will increase the pressure by a lot.

(ii) $F'(x) = -1/x^2$ so $F'(5) = -1/25$. This means that if we are five inches to the right of the endpoint, moving one more inch to the right should decrease the pressure by about 1/25 of a pound per square inch.

(b) Suppose the vertical position of a weight on a spring in inches is given as a function of time in seconds by the formula $h(t) = \cos(2t)$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $p'(t) = -2\sin(2t)$ so the velocity is zero when $\sin(2t) = 0$. This happens when $2t = 0, \pi, 2\pi, \dots$, and thus when $t = 0, \pi/2, \pi, 3\pi/2, \dots$. In other words, at $n\pi/2$.

(ii) $p''(t) = -4\cos(2t)$ is zero when $2t = \pi/2, 3\pi/2, \dots$, and thus when $t = \pi/4, 3\pi/4, 5\pi/4, \dots$. We could say $t = (2n+1)\pi/4$.