Math 1231: Single-Variable Calculus 1

George Washington University Fall 2025

Recitation 6

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Problem 1. Suppose a particle has height as a function of time given by $h(t\mathrm{s}) = (2t^3 - 3t^2 - 12t + 3)\,\mathrm{m}$.

(a) What is the velocity of this particle at time $t = 0$? What are the units, and why?

(b) What is the acceleration of this particle at time $t = 0$? What are the units and why?

(c) When is the particle speeding up? When is it slowing down?

Problem 2. Suppose that $p(t) = 10 - 2t$ is momentum (in kg m/s) of a ball thrown directly upwards, as a function of time (in seconds).

(a) What units does the derivative $p'(t)$ take as input? What units are its output? (Do you know of any physical quantity that's represented by those units?)

(b) What does the derivative $p'(t)$ represent physically? What would it mean for $p'(t)$ to be big, or small?

(c) Calculate $p'(3)$. What does this tell you? What physical observation could you measure to check if your calculation was correct?

Problem 3. Suppose the cost of buying m machines is $C(m) = 500 + 10m + .05m^2$. There's some start-up cost to having any machines at all; then each machine costs a bit more than the previous one.

(a) What are the units of the inputs to the function C? What are the units of the outputs?

(b) What is $C(1)$? $C(10)$? $C(100)$?

(c) Find a formula for $C'(m)$. What are the units of the input and output to $C'(m)$?

(d) What is $C'(10)$? How should we interpret this number?

(e) What is the *average* cost per machine when you have ten machines? How does this compare to your previous answer?

(f) What is $C''(m)$? What are the units? What is $C''(10)$ and how should we interpret it?

Problem 4 (Bonus)**.** Let $Q(p) = 10000 - 10p$ give the number of widgets you can sell at a given price p.

(a) If you set a price of \$100, how many widgets will you be able to sell? What if you set a price of \$1000?

(b) What is the derivative of Q? What are its units?

(c) What is $Q'(100)$ and what does that tell you?

Problem 5. Find an equation for the tangent line to $y = 6\cos x$ at $(\pi/3, 3)$.

Problem 6. Find an equation for the tangent line to $y\cos(x) = 1 + \sin(xy)$ at the point $(0, 1)$.

Problem 7. Suppose we have some function f such that $8f(x) + x^2(f(x))^3 = 24$, and we know that $f(4) = 1$. (Say we've measured this experimentally and now want to understand or compute with the function). Now suppose we want to estimate $f(5)$ (without having to solve the equation).

(a) Use implicit differentiation on the equation $8f(x) + x^2(f(x))^3 = 24$ to find a formula relating $x, f(x)$ and $f'(x)$.

(b) Use this formula to determine $f'(4)$.

(c) Find a formula for the linear approximation of f near 4.

(d) Estimate $f(5)$.

Problem 8 (Bonus)**.** (a) If $\sqrt{xy} = x^2y - 2$, find a formula for $\frac{dy}{dx}$ in terms of x and y.

(b) Find an equation of the tangent line at the point $(1, 4)$.