Math 1231 Fall 2025
Single-Variable Calculus I Section 11
Mastery Quiz 7
Due Monday, October 13

This week's mastery quiz has four topics. Everyone should submit S5 and S6. If you have a 4/4 on M2 you don't need to submit it again; this is your last quiz on M2. If you have a 2/2 on S4 you don't need to submit it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 4: Rates of Change
- Secondary Topic 5: Implicit Differentiation
- Secondary Topic 6: Related Rates

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx}\csc^{7/3}\left(\dfrac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)$.

Solution:

$$\frac{7}{3}\csc^{4/3}\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\left(-\csc\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\cot\left(\frac{x^2+\sin(x)}{\sqrt{x}-\cot(x)}\right)\right)$$
$$\cdot\frac{(2x+\cos(x))(\sqrt{x}-\cot(x))-\left(\frac{1}{2}x^{-1/2}+\csc^2(x)\right)(x^2+\sin(x))}{(\sqrt{x}-\cot(x))^2}.$$

(b) Compute $\dfrac{d}{dx}\left(\dfrac{x^3+\sqrt[5]{x}-3}{\sec(x^2)+2}\right)^2$.

Solution:

$$\frac{d}{dx}\left(\frac{x^3+\sqrt[5]{x}-3}{\sec(x^2)+2}\right)^2$$
$$=2\frac{x^3+\sqrt[5]{x}-3}{\sec(x^2)+2}\cdot\frac{(3x^2+\frac{1}{5}x^{-4/5})(\sec(x^2)+2)-\sec(x^2)\tan(x^2)2x(x^3+\sqrt[5]{x}-3)}{(\sec(x^2)+2)^2}.$$

Secondary Topic 4: Rates of Change

(a) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula $d(t)=t^3+4t^2+5t+4$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Solution:

(i) $d'(t)=3t^2+8t+5=(3t+5)(t+1)$ so the velocity is zero when $t=-1,-5/3$.

(ii) $d''(t)=6t+8$ is zero when $t=-4/3$.

(b) The force a magnet exerts on a piece of iron depends on the distance between the magnet and the metal. Let $F(d)=\frac{2}{d^2}$ give the force exerted by the magnet in Newtons, where d is the distance between them in meters.

(i) What are the units of $F'(d)$? What does it $F'(d)$ represent physically? What would it mean if $F'(d)$ is big?

(ii) Calculate $F'(2)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution:

(i) The derivative is the rate at which the amount of force changes as you change the distance between the magnet and the iron; its units are Netwons per meter. If $F'(d)$ is big, that means that moving the magnet a little bit will change the force on it by a lot.

(ii) $F'(d) = \frac{-4}{d^3}$ so $F'(3) = \frac{-4}{8} = -1/2$. This means that moving the iron another meter away from the magnet should reduce the force by about half a Newton.

Secondary Topic 5: Implicit Differentiation

(a) Find a formula for $\frac{d^2y}{dx^2}$ in terms of x and y if $\sin(xy) = x + y$.

Solution: Using implicit differentiation, we have

$$\begin{aligned} \cos(xy)(y + xy') &= 1 + y' \\ y\cos(xy) - 1 &= (1 - \cos(xy)x)y' \end{aligned}$$

$$\begin{aligned} y' &= (\frac{y\cos(xy) - 1}{1 - \cos(xy)x} \\ y'' &= \frac{\left(y'\cos(xy) - y\sin(xy)(y + xy')\right)(1 - \cos(xy)x) - (y\cos(xy) - 1)(x\sin(xy)(y + xy') - \cos(xy))}{(1 - \cos(xy)x)^2} \\ &= \frac{\left(\left(\frac{y\cos(xy)-1}{1-\cos(xy)x}\right)\cos(xy) - y\sin(xy)(y + x\left(\frac{y\cos(xy)-1}{1-\cos(xy)x}\right))\right)(1 - \cos(xy)x) - (y\cos(xy) - 1)(x\sin(xy}{(1 - \cos(xy)x)^2} \end{aligned}$$

(b) Find an equation for the line tangent to the curve $3x^2y + 5xy^2 = 2x$ at the point $(1, -1)$.

Solution:

$$\begin{aligned}6xy + 3x^2y' + 5y^2 + 10xyy' &= 2\\ -6 + 3y' + 5 - 10y' &= 2\\ -7y' &= 3\\ y' &= -3/7\end{aligned}$$

and thus an equation for the tangent line is

$$y + 1 = \frac{-3}{7}(x-1).$$

Secondary Topic 6: Related Rates

(a) The surface area of a cube is given by the formula $A = 6s^2$ where s is the length of a side. If the side lengths are increasing by 2 inches per second, how fast is the surface area increasing when the area is 54 square inches?

Solution: We have the data $A = 6s^2, A = 54, s' = 2$. We take a derivative and see that $A' = 12ss'$, so we need to find s. But when $A = 54$ we have

$$\begin{aligned}54 &= 6s^2\\ 9 &= s^2\\ 3 &= s\end{aligned}$$

and thus

$$A' = 12ss' = 12 \cdot 3 \cdot 2 = 72$$

so the area is increasing at 72 square inches per second.

A rocket is taking off with a perfectly vertical path, and is being tracked by a radar station on the ground four miles from the launch pad. We want to know how fast the rocket is rising when it is three miles high and its distance from the radar station is increasing at a rate of 3000 miles per hour.

(a) Choose an equation to use for this problem, and explain why you chose that equation.

(b) Use calculus to answer the question. Make sure you answer with a complete sentence.

Solution: We know one speed and want to know another, and we also know distances. This means we probably want to use the distance formula and take its derivative to find speeds.

We write $h = 3\text{mi}$, and can work out that $d = 5\text{mi}$. We know from the text of the problem that $d' = \text{mi/h}$.

We know that $d^2 = h^2 + 4^2\text{mi}^2$ and thus $2dd' = 2hh'$. Plugging in values gives us

$$\begin{aligned} 2 \cdot 5\text{mi} \cdot 3000\text{mi/hr} &= 2 \cdot 3\text{mi} \cdot h' \\ h' &= 5000\text{mi/hr}. \end{aligned}$$

Thus the rocket is rising at 5000 miles per hour.