Math 1231 Fall 2025
Single-Variable Calculus I Section 12
Mastery Quiz 7
Due Wednesday, October 15

This week's mastery quiz has three topics. Everyone should submit S5 and S6. If you have a 4/4 on M2 you don't need to submit it this week. (Check Blackboard for your current scores!) This week is the last week for M2.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Secondary Topic 5: Implicit Differentiation
- Secondary Topic 6: Related Rates

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Compute $\dfrac{d}{dx}g(x) = \left(\dfrac{x\csc(x)}{\sqrt{x^3-x}}\right)^3$

Solution:

$$g'(x) = 3\left(\frac{x\csc(x)}{\sqrt{x^3-x}}\right)^2 \frac{\big(\csc(x) - x\csc(x)\cot(x)\big)\sqrt{x^3-1} - x\csc(x)\frac{1}{2}(x^3-x)^{-1/2}(3x^2-1)}{x^3-x}.$$

Compute $\dfrac{d}{dx}\cos\left(\sec^2\left(x^3\tan(x^2)\right)\right)$.

Solution:

$$f'(x) = -\sin\left(\sec^2\left(x^3\tan(x^2)\right)\right)\cdot 2\sec\left(x^3\tan(x^2)\right)\cdot\sec\left(x^3\tan(x^2)\right)\tan\left(x^3\tan(x^2)\right)$$
$$\cdot\left(3x^2\tan(x^2) + x^3\sec^2(x^2)\cdot 2x\right)$$

Secondary Topic 5: Implicit Differentiation

(a) Find an equation for the line tangent to the curve $x^2y - xy^3 = xy + 3$ at the point $(3, 1)$.

Solution:

$$\begin{aligned} 2xy + x^2y' - y^3 - 3xy^2y' &= y + xy' \\ 6 + 9y' - 1 - 9y' &= 1 + 3y' \\ 4 &= 3y' \\ y' &= 4/3 \end{aligned}$$

and thus an equation for the tangent line is

$$y - 1 = \frac{4}{3}(x-3).$$

(b) Find a formula for $\frac{d^2y}{dx^2}$ if $x^3 = xy + 1$.

Solution:

$$
\begin{aligned}
3x^2 &= y + xy' \\
y' &= \frac{3x^2 - y}{x} && = 3x - \frac{y}{x} \\
y'' &= \frac{(6x - y')x - (3x^2 - y)}{x^2} && = 3 - \frac{y'x - y}{x^2} \\
&= \frac{\left(6x - \frac{3x^2 - y}{x}\right)x - (3x^2 - y)}{x^2} && = 3 - \frac{\frac{y}{x} \cdot x - y}{x^2} = 3.
\end{aligned}
$$

Secondary Topic 6: Related Rates

(a) A snowball is melting such that its surface area is decreasing at $1\text{cm}^2/\text{min}$. When the radius is 8cm, how quickly is the radius decreasing?

(The surface area of a sphere of radius r is $4\pi r^2$.)

(a) Choose an equation to use for this problem, and explain why you chose that equation.

(b) Use calculus to answer the question. Make sure you answer with a complete sentence.

Solution: We have $S = 4\pi r^2$, so $S' = 8\pi r r'$. When the radius is 8cm we have

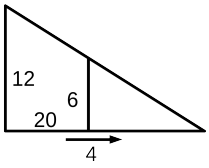
$$
\begin{aligned}
S' &= 8\pi \cdot 8\text{cm} \cdot r' \\
-1\text{cm}^2/\text{min} &= 64\pi r' \\
r' &= \frac{-1}{64\pi}\text{cm}/\text{min}.
\end{aligned}
$$

Thus when the radius is 8cm, the radius of the snowball is decreasing by $\frac{1}{64\pi}$ centimeters per minute.

(b) A street light is mounted at the top of a 12-foot-tall pole. A six-foot-tall man walks straight away from the pole at 4 feet per second. We want to know how fast the length of his shadow is changing when he is twenty feet from the pole.

(a) Choose an equation to use for this problem, and explain why you chose that equation.

(b) Use calculus to answer the question. Make sure you answer with a complete sentence.



Solution: After drawing a picture, we see we have two triangles in the same shape: we know how one triangle is changing, and we want to figure out how the other is changing, so we should relate those similar triangles.

Let d be the distance of the man from the pole. Then $d = 20$ and $d' = 4$. If s is the length of the shadow, then we have $s/6 = (d + s)/12$ so we get

$$\begin{aligned} s &= \frac{d+s}{2} \\ s' &= d'/2 + s'/2 \\ s'/2 &= d'/2 \\ s' &= d' = 4. \end{aligned}$$

Thus the length of the shadow is growing at 4 feet per second.