Math 1231: Single-Variable Calculus 1
George Washington University Fall 2025
Recitation 7

Jay Daigle

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Problem 1. Find an equation for the tangent line to $y = 6\cos x$ at $(\pi/3, 3)$.

Problem 2. Find an equation for the tangent line to $y\cos(x) = 1 + \sin(xy)$ at the point $(0, 1)$.

Problem 3. Suppose we have some function f such that $8f(x) + x^2(f(x))^3 = 24$, and we know that $f(4) = 1$. (Say we've measured this experimentally and now want to understand or compute with the function). Now suppose we want to estimate $f(5)$ (without having to solve the equation).

(a) Use implicit differentiation on the equation $8f(x) + x^2(f(x))^3 = 24$ to find a formula relating $x, f(x)$ and $f'(x)$.

(b) Use this formula to determine $f'(4)$.

(c) Find a formula for the linear approximation of f near 4.

(d) Estimate $f(5)$.

Problem 4 (Bonus)**.** (a) If $\sqrt{xy} = x^2y - 2$, find a formula for $\frac{dy}{dx}$ in terms of x and y.

(b) Find an equation of the tangent line at the point $(1, 4)$.

Problem 5 (Bonus)**.** A rectangle is getting longer by one inch per second and wider by two inches per second. When the rectangle is 5 inches long and 7 inches wide, how quickly is the area increasing?

(a) Draw a picture of this situation.

(b) What is the question you're trying to answer? What do you expect it to look like? Should it be positive or negative? What units do you expect?

(c) What equation should we use here, and why?

(d) Use a derivative to calculate the answer to the question. Does your answer make sense?

(e) To check things: how long and wide will the rectangle be after one inch? How much will the area have increased? Does that make sense with your answer to the related rates problem?

(f) Bonus: where have we seen basically this argument before?