

Math 1231: Single-Variable Calculus 1
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Recitation 7

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Problem 1. A twenty foot ladder rests against a wall. The bit on the wall is sliding down at 1 foot per second. How quickly is the bottom end moving when the top is 12 feet from the ground?

- (a) Draw a picture of this situation.
- (b) What is the question you're trying to answer? What do you expect it to look like? Should it be positive or negative? What units do you expect?
- (c) What equation should we use here, and why?
- (d) Use a derivative to calculate the answer to the question. Does your answer make sense?

Solution: Let h be the height of the ladder on the wall, and b be the distance of the foot of the ladder from the wall. Then $h = 12$, $h' = -1$, and $b = \sqrt{400 - 144} = 16$. We have

$$h^2 + b^2 = 400$$

$$2hh' + 2bb' = 0$$

$$2 \cdot 12 \cdot (-1) + 2 \cdot 16 \cdot b' = 0$$

$$b' = \frac{24}{32} = 3/4$$

so the foot of the ladder is sliding away from the wall at 3/4 ft/s. Again, the direction of the sliding is correct (away from the wall), and the number seems plausible.

Problem 2. A spot light is on the ground 36 ft away from a wall and a 5 ft tall person is walking towards the wall at a rate of 4 ft/sec. How fast is the height of the shadow changing when the person is 24 feet from the wall? Is the shadow increasing or decreasing in height at this time?

- (a) Draw a picture of this situation.
- (b) What is the question you're trying to answer? What do you expect it to look like? Should it be positive or negative? What units do you expect?
- (c) What equation should we use here, and why?
- (d) Use a derivative to calculate the answer to the question. Does your answer make sense?

Solution:

- (a)
- (b) We want to know how fast the height of the shadow is changing. This should be in feet per second. Physically, it seems like the shadow should be shrinking.
- (c) This is a similar triangles problem, because we can compare the triangle with height given by the person, to a triangle with height given by the shadow.
- (d) Let h be the height of the shadow, and d be the distance between the wall and the person. Then we want to find h' . We currently have $d = 24$. We know by similar triangles that $\frac{36-d}{36} = \frac{5}{h}$, which tells us that currently $h = 15$.

Then we have $d' = -4$. We compute

$$\begin{aligned}\frac{-d'}{36} &= \frac{-5h'}{h^2} \\ \frac{1}{9} &= \frac{-h'}{45} \\ h' &= \frac{-45}{9} = -5.\end{aligned}$$

Thus the shadow's height is decreasing by 5 feet per second.

Problem 3. Consider the function $f(x) = x^3 - 3x^2 + 1$ on $[-1, 4]$.

- (a) Does this function have absolute extrema? Why?
- (b) What are the critical points of this function?
- (c) How many absolute extrema are there? What are they, and where are they?

Solution:

- (a) This function is continuous on a closed interval, so by the Extreme Value Theorem it must have an absolute maximum and an absolute minimum.
- (b) $f'(x) = 3x^2 - 6x$ and is defined everywhere. We have $3x^2 - 6x = 0$ when $x = 0$ or $x = 2$, so the critical points are 0 and 2.
- (c) We need to evaluate all the possible extrema: the critical points and the endpoints. We compute $f(-1) = -3$, $f(0) = 1$, $f(2) = -3$, $f(4) = 17$. Thus the absolute maximum is 17 at 4, and the absolute minimum is -3 at -1 and 2.

Problem 4. Let's find the global extrema of $g(x) = \sqrt[3]{x^3 + 6x^2}$ on the closed interval $[-5, 5]$.

- (a) Does this function have absolute extrema? Why?
- (b) What are the critical points of this function?
- (c) How many absolute extrema are there? What are they, and where are they? (Hint: you may need to use a calculator at the last step.)

Solution:

- (a) This function is continuous on a closed interval, so by the Extreme Value Theorem it must have an absolute maximum and an absolute minimum.
- (b) We take the derivative, and compute

$$g'(x) = \frac{1}{3}(x^3 + 6x^2)^{-2/3}(3x^2 + 12x) = \frac{3x(x+4)}{3\sqrt[3]{(x^3 + 6x^2)^2}}.$$

This derivative is zero when $x = -4$, and it doesn't exist when $x = 0$ or $x = -6$. (You might think that $g'(0) = 0$, but it's actually just undefined.) The critical points are $-6, -4, 0$, but we can ignore the case where $x = -6$, since it's not inside the interval.

- (c) We need to evaluate all the possible extrema: the critical points inside the interval, and the endpoints. We might need a calculator here, but we

$$g(-5) = \sqrt[3]{-125 + 150} = \sqrt[3]{25} \approx 2.9$$

$$g(-4) = \sqrt[3]{-64 + 96} = \sqrt[3]{32} \approx 3.17$$

$$g(0) = \sqrt[3]{0} = 0$$

$$g(5) = \sqrt[3]{125 + 150} = \sqrt[3]{275} \approx 6.5.$$

Thus the absolute minimum is 0, which occurs at 0, and the absolute maximum is $\sqrt[3]{275} \approx 6.5$, which occurs at 5.

Note that if you forget about the critical point where $g'(x)$ is undefined, you will miss the minimum! The minimum is very definitely not $\sqrt[3]{25}$, which you can see if you graph the function.