Math 1232 Practice Final Solutions

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- These are the instructions you will see on the real test, next week. I include them here so you know what to expect.
- You will have 120 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made for yourself ahead of time. You must have written on the physical sheet you bring to the test in your own handwriting.
- You may not use a calculator.
- The exam has 14 problems, one on each mastery topic we've covered. The exam has 11 pages total.
- On the real final, each major topic will two questions, worth 10 points each. On this practice final I have given three questions on each major topic, for extra practice. Each secondary topic is worth 10 points.
- **You may not answer all the secondary topic questions.** You may attempt up to six secondary topics. Your four best will count towards your score on the final. You may get one or two bonus points for the fifth and sixth. **We will not grade more than six secondary topics.**
- If you perform well on a question on this test it will update your mastery scores. Achieving a 18/20 on a major topic or 9/10 on a secondary topic will count as getting a 2 on a mastery quiz.

Problem 1 (M1). (a) Compute $\int \frac{x}{\sqrt{4-x^4}}\,dx$.

(b) Compute $\int 5^{3x}\,dx$.

(c) Write a tangent line to the curve $y^2 = x^{x\cos(x)}$ at the point $(\pi/2, -1)$.

Problem 2 (M2)**.** Compute the following integrals:

(a) $\displaystyle\int \sin x \cos 2x \, dx$

(b) $\displaystyle\int_{\sqrt{7}}^{2\sqrt{7}} \frac{dx}{x\sqrt{x^2-7}}$

(c) $\displaystyle\int \frac{4}{(x^2+1)(x+1)(x-1)} \, dx$

Problem 3 (M3)**.** Analyze the convergence of the following series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{3(-1)^n}{n\ln(n)}$.

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{5^n+3}{3^n-2}$.

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n^3+n^2+n+1}{\sqrt{n^9}}.$

Problem 4 (M4). (a) Find a power series for $\frac{1}{x^3}(e^{2x^3} - 1)$, and write down the first three non-zero terms explicitly.

(b) Find a power series for $x^2 \arctan(x^2)$ centered at 0.

(c) Find the degree-three Taylor polynomial for $f(x) = \frac{3}{x^3}$ centered at 3.

Problem 5 (S1)**.** Let $g(x) = \sqrt[5]{x^9 + x^7 + x + 1}$. Find $(g^{-1})'(1)$.

Problem 6 (S2)**.** Compute $\lim_{x \to 0} \dfrac{e^x - \tan(x) - 1}{x^2}$

Problem 7 (S3)**.** How many intervals do you need with the **trapezoid** rule to approximate $\int_0^3 \frac{1}{1+x}$ to within 1/2? Compute that approximation. (Feel free to use a calculator to plug in numeric values, or to leave the answer in exact unsimplified terms, but show every step.)

Problem 8 (S4)**.** $\displaystyle\int_1^{+\infty} \frac{1}{x^2 - 2x}\, dx$

Problem 9 (S5)**.** Find the area of the surface obtained by rotating the curve $x = 1 + 2y^2$ for $1 \leq y \leq 2$ about the x-axis.

Problem 10 (S6)**.** Find the (specific) solution to $y' = x^2 y^3$ if $y(0) = 1$.

Problem 11 (S7)**.** Compute $\lim_{n\to\infty} \frac{2^n n!}{(2n)!}$.

Problem 12 (S8)**.** Find the radius and interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-3)^n}{(2n)^2+1}$.

Problem 13 (S9)**.** Use a second-degree Taylor polynomial to approximate $\sqrt[4]{82}$.

Problem 14 (S10)**.** Find an equation for the tangent line to the curve defined by the polar equation $r = 2 + \sin(3\theta)$ at the point $\theta = \pi/4$.