Math 1232: Single-Variable Calculus 2
George Washington University Spring 2025
Recitation 10

Jay Daigle

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Problem 1 (Infinite Decimals)**.** We want to find a rational representation of the infinite decimal $0.\overline{47}$. That is, we want to write $0.\overline{47} = \frac{p}{q}$ for integers p, q.

(a) First, what happens if we multiply $0.\overline{47}$ by 100?

(b) Using part (a), what can you tell about $(99) \cdot 0.\overline{47}$?

(c) Give a rational representation of $0.\overline{47}$.

(d) Now let's take a different approach. Write $0.\overline{47}$ as an infinite series.

(e) What kind of series is this? Can you use that fact to find a rational representation of $0.\overline{47}$?

(f) Now use the same logic to find a rational representation of $2.\overline{63}$.

Problem 2. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^4}$.

(a) Can you find a simpler series that's larger than this? One that's smaller?

(b) Do the series you found in part (a) converge or diverge? Why? What does that tell you about our original series?

(c) Now consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^4}$. Can you find a simpler series that's larger than this? One that's smaller?

(d) Do the series you found in part (c) converge or diverge? What does that tell you about the convergence of the original series?

(e) Now let's not worry about bigger or smaller. What is the simpler series that part (c) *kinda looks like*?

(f) How can we use that to figure out if the series in part (c) converges or diverges?

Problem 3. For each of the following series, write a careful argument showing either that it converges or that it diverges. Think about exactly what test you want to use and why.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{5n^3-2}{3n^5-n}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3\ln(n)+1}{n^4-7}$.

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

Problem 4 (Bonus)**.** Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n^2+e^n)}{n^2}$ converge or diverge?