

Math 1232: Single-Variable Calculus 2
George Washington University Spring 2025
Recitation 12

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April 9, 2025

Problem 1. Consider the function $f(x) = \frac{1}{1+x^6}$.

- (a) Could you compute $\int \frac{1}{1+x^6} dx$? How?
- (b) Does it help if I tell you that $1+x^6 = (1+x^2)(x^2 - \sqrt{3}x + 1)(x^2 + \sqrt{3}x + 1)$?
- (c) Now write a power series for $f(x)$ centered at 0. What is the interval of convergence?
- (d) Compute the integral of your power series. What is the interval of convergence there?

Problem 2. We want to compute $\int_3^4 \frac{1}{1-(x-4)^3} dx$

- (a) Find a power series for to compute $\frac{1}{1-(x-4)^3}$. Where is it centered? Will it converge from 3 to 4?
- (b) Find an antiderivative for this power series. Where will it converge?
- (c) Now compute the integral from 3 to 4. You should get a series. Does it converge? How could we predict that?
- (d) Sum the first five terms with a calculator to estimate $\int_3^4 \frac{1}{1-(x-4)^3} dx$.
- (e) Use an online integral calculator to find the integral. How close is your answer to the true answer?
- (f) What would you predict about $\int_3^5 \frac{1}{1-(x-4)^3} dx$?

Problem 3. Let's find the Taylor series of $f(x) = e^x$ centered at $a = 1$.

- (a) Compute f', f'', f''' . Find a formula for $f^{(n)}(x)$.
- (b) Give a formula for $T_f(x, 1)$.
- (c) We want to know if $f(x) = T_f(x, 1)$. Find a formula for $R_k(x, 1)$. Can you show this goes to 0 as k goes to infinity?
- (d) We already have another power series for f :

$$T_f(x, 0) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

You should have a different power series; but can you convince yourself it *should* give the same function? (What happens if you plug $x - 1$ into this series?)

Problem 4. Let's do something silly, and compute the Taylor series of a polynomial.

- (a) Let $f(x) = x^3 + 3x^2 + 1$. Find the Taylor series centered at zero. Was that what you expected?
- (b) Now find the Taylor series centered at 2. Do you get the same thing? What's useful about this?

Problem 5. Back in section 2 we talked about the bell curve function $p(x) = e^{-x^2}$. (Technically we should be talking about $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ but that's annoying and doesn't change the details enough to be interesting.)

- (a) Find a power series for $p(x)$ centered at zero. (This should not require any real calculations.)
- (b) Find an antiderivative for $p(x)$, using power series.
- (c) Write down a series that computes $\int_0^1 p(x) dx$.
- (d) Add up the first three or four terms of this series. What do you get? Can you estimate the error in this calculation?