Math 1232 Spring 2025
Single-Variable Calculus 2
Mastery Quiz 2
Due Thursday, January 30

This week's mastery quiz has two topics. Everyone should submit M1. If you got a 2/2 on S1 last week (check Blackboard!) you don't need to submit it again; but if you have a 1/2 or 0/2 you should try again. (Please read last week's solutions to see how you can improve!)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Transcendental Functions
- Secondary Topic 1: Invertible Functions

Name:

Recitation Section:

M1: Transcendental Functions

(a) Compute $\frac{d}{dx}\left(\sqrt{x+1}\right)^x$

Solution:

$$\begin{aligned} y &= \sqrt{x+1}^x \\ ln|y| &= x\ln(\sqrt{x+1}) = \frac{1}{2}x\ln(x+1) \\ y'/y &= \frac{1}{2}\left(\ln(x+1) + \frac{x}{x+1}\right) \\ y' &= \frac{1}{2}\sqrt{x+1}^x\left(\ln(x+1) + \frac{x}{x+1}\right) \end{aligned}$$

(b) (Note this is a definite integral)

$\int_0^2 \frac{e^x}{e^x+1}\,dx =$

Solution: We can take $u = e^x$ so $du = e^x\,dx$ and

$$\int_0^2 \frac{e^x}{e^x+1}\,dx = \int_1^{e^2} \frac{1}{u+1}\,du = \ln|u+1|\Big|_1^{e^2} = \ln(e^2+1) - \ln(2).$$

(c) $\int \frac{x}{9+x^4}\,dx =$

Solution: We can factor a 9 out to get $\frac{1}{9}\frac{x}{1+x^4/9}$. Then we set $u = x^2/3$, and $du = 2x/3\,dx$, and we have

$$\begin{aligned} \int \frac{x}{9+x^4}\,dx &= \int \frac{1}{9}\frac{x}{1+u^2}\frac{3}{2x}\,du \\ &= \int \frac{1}{6}\frac{1}{1+u^2}\,du \\ &= \frac{1}{6}\arctan u + C = \frac{1}{6}\arctan(x^2/3) + C. \end{aligned}$$

S1: Invertible Functions

(a) Find a formula for the inverse of $g(x) = (x^3+3)^3$.

Solution:

$$\begin{aligned} y &= (x^3+3)^3 + \sqrt[3]{y} & &= x^3+3 \\ \sqrt[3]{y} - 3 &= x^3 \\ x &= \sqrt[3]{\sqrt[3]{y}-3} \end{aligned}$$

so $g^{-1}(y) = \sqrt[3]{\sqrt[3]{y}-3}$. (You can use whichever variable you like in your formula.)

(b) Give an exact solution (with no decimals) for the equation $\ln(2x+5) = 3$.

Solution:

$$\begin{aligned} \ln(2x+5) &= 3 \\ 2x+5 &= e^3 \\ 2x &= e^3 - 5 \\ x &= \frac{e^3-5}{2}. \end{aligned}$$

(c) Let $h(x) = e^{x^3+x}$. Compute $(h^{-1})'(e^2)$.

Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(e^2) = \frac{1}{h'(h^{-1}(e^2))}.$$

Guess and check shows that $h(1) = e^2$ so $h^{-1}(e^2) = 1$. And we know that

$$h'(x) = e^{x^3+x}(3x^2+1)$$

and thus

$$h'(1) = e^2(3+1) = 4e^2.$$

Thus

$$(h^{-1})'(5) = \frac{1}{4e^2}.$$