Math 1232 Spring 2025
Single-Variable Calculus 2
Mastery Quiz 3
Due Thursday, February 6

This week's mastery quiz has two topics. Everyone should submit both of them. (Even if you got a 2 on M1 last week, you need to get two 2s to get full credit.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Transcendental Functions
- Secondary Topic 2: L'Hospital's Rule

Name:

Recitation Section:

M1: Transcendental Functions

(a) Compute $\dfrac{d}{dx}x^{e^x}$

Solution:

$$\begin{aligned} y &= x^{e^x} \\ ln|y| &= e^x \ln|x| \\ y'/y &= e^x \ln|x| + \frac{e^x}{x} \\ y' &= e^x \ln|x| x^{e^x} + \frac{1}{x} e^x x^{e^x}. \end{aligned}$$

(b) $\displaystyle\int \frac{1}{x\sqrt{4-\ln(x)^2}}\,dx =$

Solution: We can set $u = \ln(x)/2$ so $du = \frac{1}{2x}\,dx$. Then

$$\begin{aligned} \int \frac{1}{x\sqrt{4-\ln(x)^2}}\,dx = \int \frac{2}{\sqrt{4-4u^2}}\,du &= \int \frac{1}{\sqrt{1-u^2}}\,du \\ = \arcsin(u) + C &= \arcsin(\ln(x)/2) + C. \end{aligned}$$

(c) Compute $\displaystyle\int_0^{\ln(5)} e^x\sqrt{1+3e^x}\,dx.$

Solution: Set $u = 1+3e^x$, so $du = 3e^x\,dx$, and we see that $u(0) = 4$ and $u(\ln(5)) = 16$.

$$\begin{aligned} \int_0^{\ln(5)} e^x\sqrt{1+3e^x}\,dx &= \int_4^{16} \frac{1}{3}\sqrt{u}\,du \\ &= \frac{2}{9}u^{3/2}\Big|_4^{16} = \frac{2}{9}\cdot 64 - \frac{2}{9}\cdot 8 = \frac{112}{9}. \end{aligned}$$

Alternatively, we could substitute our integral back:

$$\begin{aligned} \int e^x\sqrt{1+3e^x}\,dx &= \int \frac{1}{3}\sqrt{u}\,du \\ &= \frac{2}{9}u^{3/2} + C = \frac{2}{9}(1+3e^x)^{3/2} + C. \\ \int e^x\sqrt{1+3e^x}\,dx &= \frac{2}{9}(1+3e^x)^{3/2}\Big|_0^{\ln(5)} \\ &= \frac{2}{9}\cdot 64 - \frac{2}{9}\cdot 8 = \frac{112}{9}. \end{aligned}$$

S2: L'Hospital's Rule

(a) $\displaystyle\lim_{x\to-\infty} \frac{e^x}{\arctan(x)+\pi/2} =$

Solution:

$$\begin{aligned}\lim_{x\to-\infty} \frac{e^x}{\arctan(x)+\pi/2} &= \lim_{x\to-\infty} \frac{e^x}{\frac{1}{x^2+1}} = \lim_{x\to-\infty} x^2e^x + e^x = \lim_{x\to-\infty} x^2e^x \\ &= \lim_{x\to-\infty} \frac{x^2}{e^{-x}} = \lim_{x\to-\infty} \frac{2x}{-e^{-x}} \\ &= \lim_{x\to-\infty} \frac{2}{e^{-x}} = 0.\end{aligned}$$

(b) $\displaystyle\lim_{x\to 0} \frac{x^3-x^2}{x+\sin(x)} =$

Solution: The limits of the top and bottom are both zero, so we can use L'Hospital's Rule:

$$\lim_{x\to 0} \frac{x^3-x^2 \nearrow^0}{x+\sin(x)\searrow_0} \stackrel{\text{L'H}}{=} \lim_{x\to 0} \frac{3x^2-2x \nearrow^0}{1+\cos(x)\searrow_2} = \frac{0}{2} = 0.$$

Note we *cannot* use L'Hospital's rule a second time, because we don't have an indeterminate form.

(c) $\displaystyle\lim_{x\to 2} \frac{e^{(x^2-4)}-x+1}{x-2} =$

Solution: The limit of the top and bottom are both 0, we can use L'Hospital's rule.

$$\lim_{x\to 2} \frac{e^{x^2-4}-x+1 \nearrow^0}{x-2\searrow_0} \stackrel{\text{L'H}}{=} \lim_{x\to 2} \frac{2xe^{x^2-4}-1}{1} = 3.$$