Math 1232: Single-Variable Calculus 2
George Washington University Spring 2025
Recitation 6

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Problem 1. We want to find the cross-sectional area of a two-meter-long airplane wing. We measure its width every 20 centimeters, and get: 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, 2.8. Use the trapezoidal rule and Simpson's rule to estimate the area of the wing.

Problem 2. Consider the function $f(x) = x^2 + 1$.

(a) Use the trapezoid rule with six intervals to estimate $\int_{-4}^{2} f(x)\,dx$.

(b) Use the midpoint rule with six intervals to estimate $\int_{-4}^{2} f(x)\,dx$.

(c) Use Simpson's rule with six intervals to estimate $\int_{-4}^{2} f(x)\,dx$.

(d) Which of these do you expect to be most accurate? Which do you expect to be least accurate?

(e) Compute $\int_{-4}^{2} f(x)\,dx$. What do you find? Why?

Problem 3. Let $g(x) = e^{-x^2}$, and suppose we want to compute $\int_{-1}^{2} e^{-x^2}\,dx$, and get the answer correct to two decimal places.

(a) We can compute that $g''(x)$ varies between -2 and $.9$ when x is in $[-1, 2]$. What value should we take for K?

(b) How many subintervals should we use to get the answer correct to within two decimal places using the trapezoid rule?

(c) How many subintervals should we use to get the answer correct to within two decimal places using the midpoint rule?

(d) We can compute that $g''''(x)$ varies between -8 and 12. What value should we take for L?

(e) How many subintervals should we use to get the answer correct to within two decimal places using Simpson's rule?

Problem 4. We want to compute $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\, dx$.

(a) Can you compute an antiderivative? Can you evaluate it at 0 and 2?

(b) Did part (a) finish the problem? Sketch a picture of the graph. What should we be concerned about?

(c) Carefully set up a computation that will find $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\, dx$. (Hint: It should have two limit operations in it.)

(d) What did we learn from this that we didn't learn from (a)?

Problem 5 (Bonus)**.** We want to figure out if $\int_0^{+\infty} e^{-x^2}\, dx$ converges—that is, if it's finite or infinite.

(a) If we can find an antiderivative, we can just compute the improper integral directly. Why doesn't that work?

(b) Since we can't integrate this directly we might want to use a comparison test. We need to find an easy-to-integrate function that's larger than e^{-x^2}. Find a function $f(x)$ that makes $f(x)e^{-x^2}$ easy to integrate.

(c) If $f(x) \geq 1$, then we can just integrate $f(x)e^{-x^2}$. Is it?

(d) This is where we can pull in a trick. Is there some a where $f(x) > 1$ when $x > a$? (You may need to adjust your $f(x)$ here, especially the sign. It's fine as long as you can still integrate it.)

(e) We know $\int_a^{+\infty} e^{-x^2}\, dx \leq \int_a^{+\infty} f(x)e^{-x^2}\, dx$. Compute the new improper integral; is it finite?

(f) Now we just have to deal with $\int_0^a e^{-x^2}\,dx$. We can't do that integral exactly, but that's fine: you should be able to tell whether it's finite or not without doing any calculations. How?

(g) Does $\int_0^{+\infty} e^{-x^2}\,dx$ converge?