Math 1232: Single-Variable Calculus 2
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Recitation 7

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Problem 1. Let $f(x) = x^2$. Let's find the arc length between $x = 0$ and $x = 4$.

(a) This makes a very reasonable shape. What does the graph look like?

(b) Set up an integral to compute this arc length. You need to think about the variable of integration, the bounds, and the actual function to integrate.

(c) What techniques should we use to compute this integral? Where do we get stuck?

(d) Is there another way we could have set it up?

(e) Is that integral any easier?

Problem 2. Let $f(x) = \sqrt[3]{3x}$. Take the portion of the graph where $0 \leq y \leq 2$ and rotate it around the y axis.

(a) Try to sketch a picture of what this will look like.

(b) Set up an integral to find the surface area. Again, think about the variable of integration, the bounds, and the function. Do you have multiple choices here or just one?

(c) Can you compute that integral?

Problem 3. Suppose we have a Hooke's Law system of a weight on a spring. Suppose $m = k$, so that we get the differential equation $x''(t) = -x(t)$.

(a) From class, we know the general solution to this differential equation. What is it?

(b) Suppose now we start (at time 0) with the weight stationary and displaced by 1 meter. What initial conditions does this correspond to?

(c) Find the specific solution to this initial value problem.

(d) What does this describe physically? Does that solution make physical sense?

Problem 4. We can also do a different setup that is not technically an initial value problem. Suppose we have a Hooke's Law setup with a weight on a spring, and $m = k$, so $x(t) = a\sin(t) + b\cos(t)$.

(a) Suppose the weight starts with a displacement of 2, and at time $t = \pi/4$ the displacement is $\sqrt{8}$. How can we encode that mathematically?

(b) Why is this not an initial value problem? (We call this a "boundary value problem". Why do you think we call it that?)

(c) Is this enough information to find a specific solution to the differential equation? What is it?