Math 1232: Single-Variable Calculus 2
George Washington University Spring 2025
Recitation 9

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March 19, 2025

Problem 1. Let $(a_n) = \left(-6, 4, \frac{-8}{3}, \frac{16}{9}, \frac{-32}{27}, \dots\right)$.

(a) Find a closed-form formula for a_n.

(b) Is there a real function f so that $f(n) = a_n$?

(c) What is $\lim_{n\to\infty} a_n$? Why?

Problem 2 (Factorials)**.** (a) What is $4!$? What is $\frac{4!}{3!}$?

(b) What is $\frac{5!}{4!}$? What is $\frac{5!}{3!}$?

(c) Can you figure out what $\frac{202!}{200!}$ is?

Problem 3. (a) Compute $\lim_{n\to\infty} \dfrac{n}{n!}$. Justify your answer.

(b) Compute $\lim_{n\to\infty} \frac{e^n}{n!}$.

(c) Now compute $\lim_{n\to\infty} \frac{n^k}{n!}$, where $k > 0$ is a fixed integer.

Problem 4. Write out the first five terms of:

(a) $\sum_{k=1}^{\infty} \frac{(-2)^{k+1}}{3k}$

(b) $\sum_{k=1}^{\infty} \frac{k+1}{k!}$

(c) $\sum_{k=3}^{\infty} \frac{k+3}{k^2-k-2}$

Problem 5. Write in series/summation notation:

(a) $1 + \frac{2}{3} + \frac{3}{5} + \frac{4}{7} + \dots$

(b) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} + \dots$

(c) $2 + 7 + 14 + 23 + 34 + \dots$

Problem 6. (a) Use a telescoping series argument to write down a formula for $\sum_{k=1}^{n} \frac{1}{k^2+3k+2}$.

(b) Compute $\sum_{k=1}^{\infty} \frac{1}{k^2+3k+2}$.

(c) Use a telescoping series argument to write down a formula for $\sum_{k=1}^{n} \frac{2}{k^2+2k}$.

(d) Compute $\sum_{k=1}^{\infty} \frac{2}{k^2+2k}$.

(e) Use a telescoping series argument to write down a formula for $\sum_{k=1}^{n} \ln\left(\frac{k+1}{k+3}\right)$.

(f) Compute $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k+3}\right)$.

Problem 7 (Geometric Series)**.** Compute:

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{3^k}$

(b) $\displaystyle\sum_{k=2}^{\infty} \frac{(-5)^{k+2}}{2^{3k}}$

(c) $\frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \frac{5}{16} + \dots$

(d) $\frac{-2}{3} + \frac{8}{9} + \frac{-32}{27} + \dots$

(e) $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \dots$