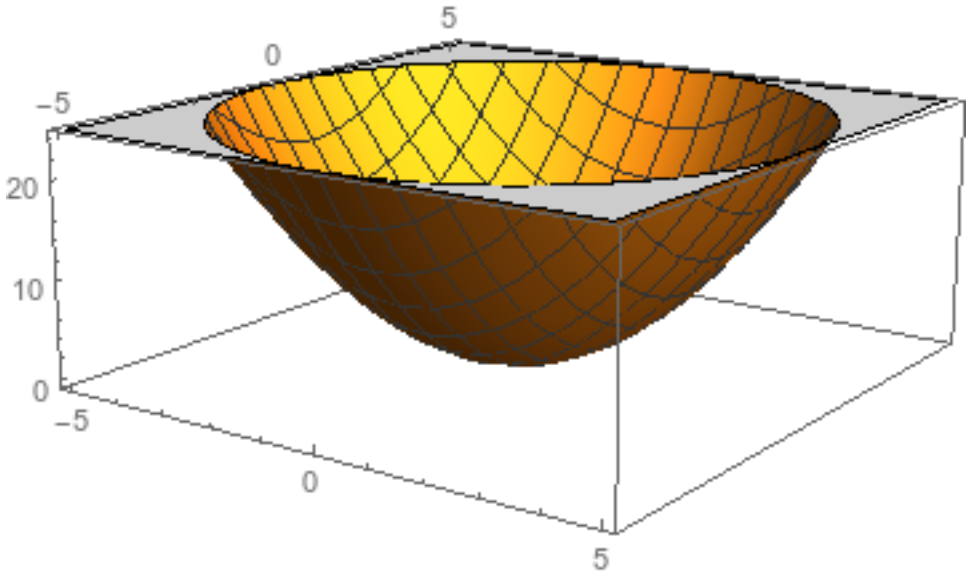
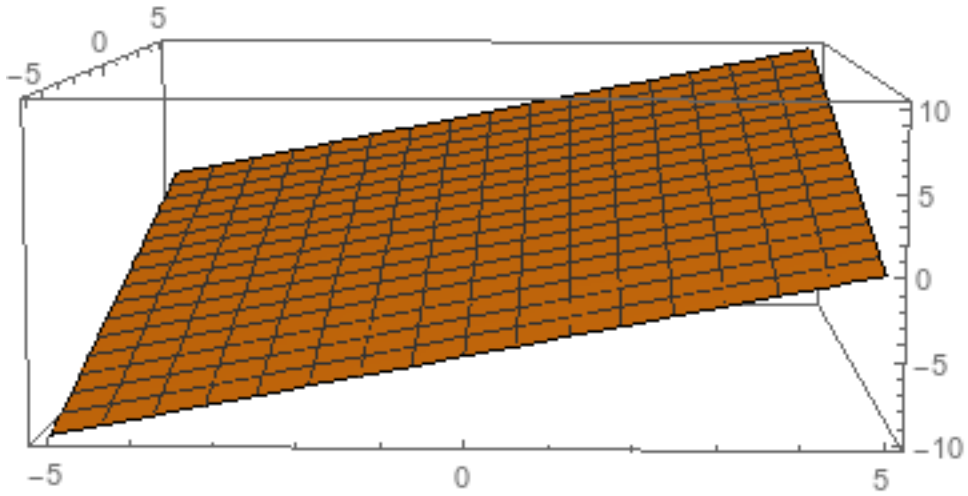
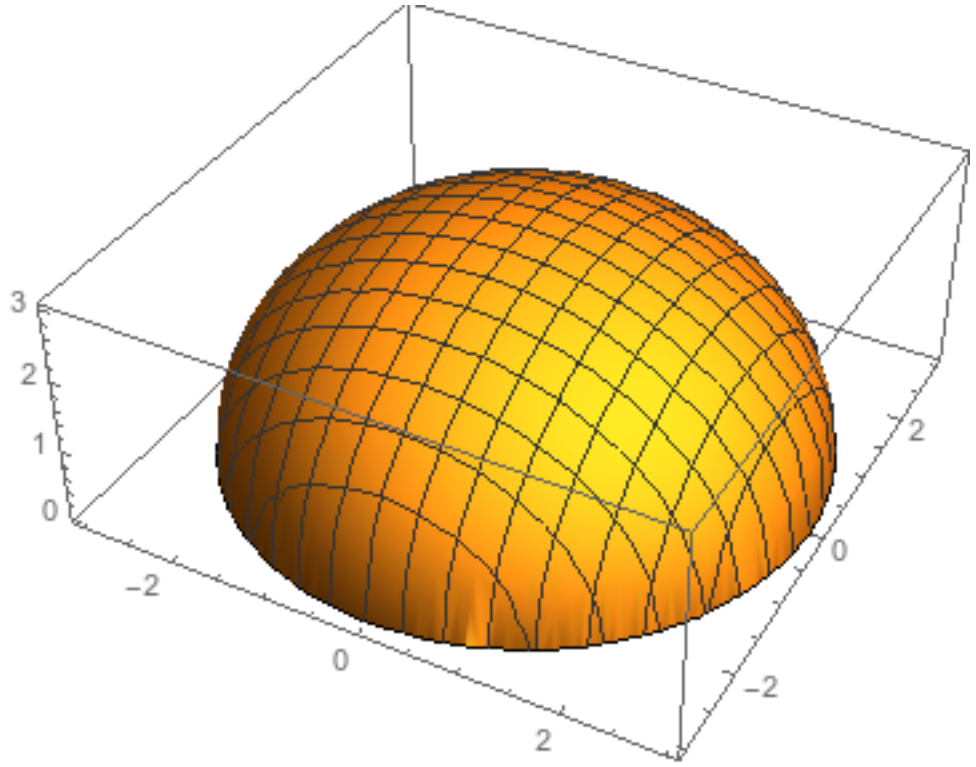
3 Derivatives

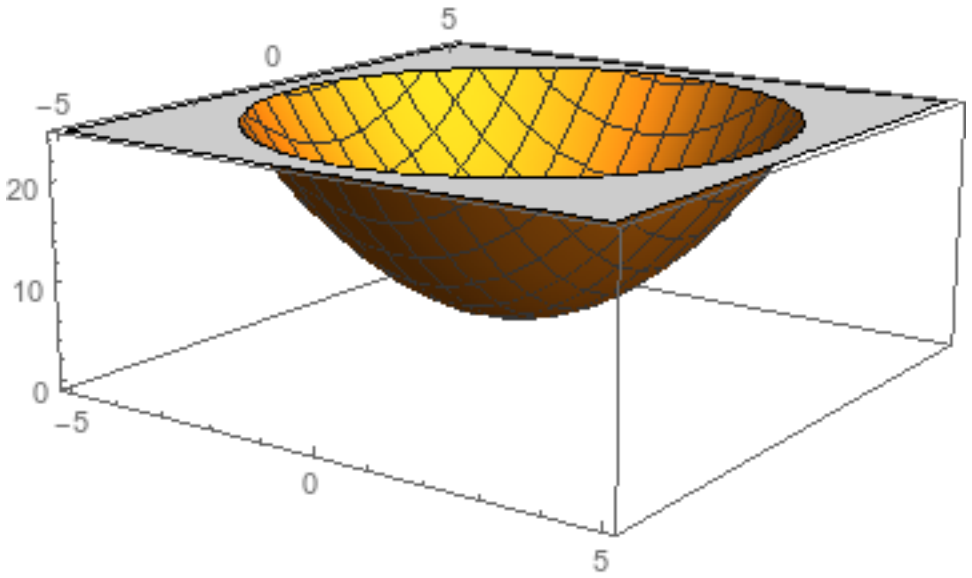
3.1 Graphing multivariable functions

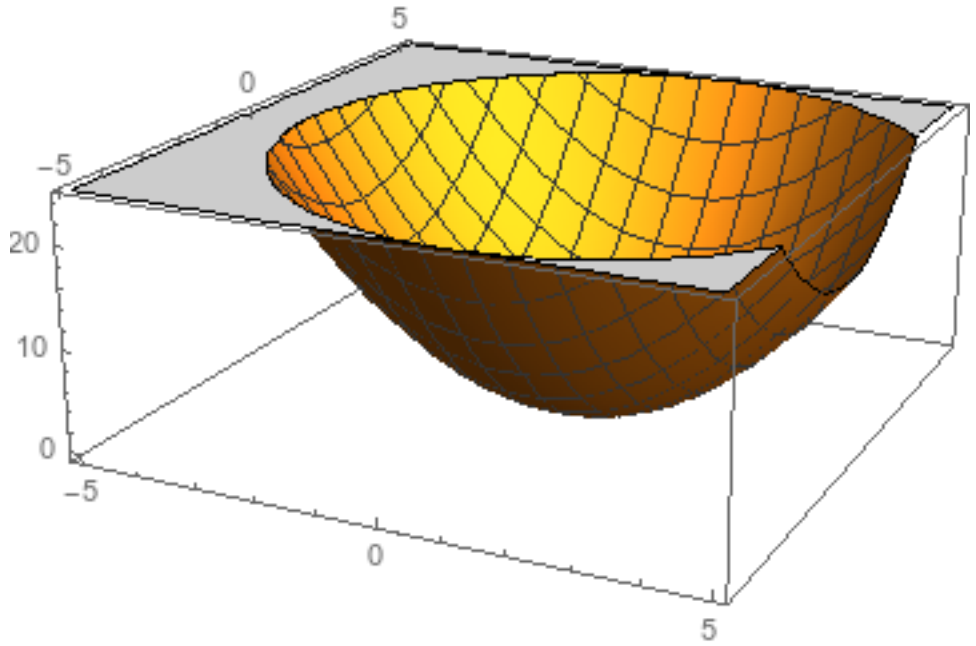


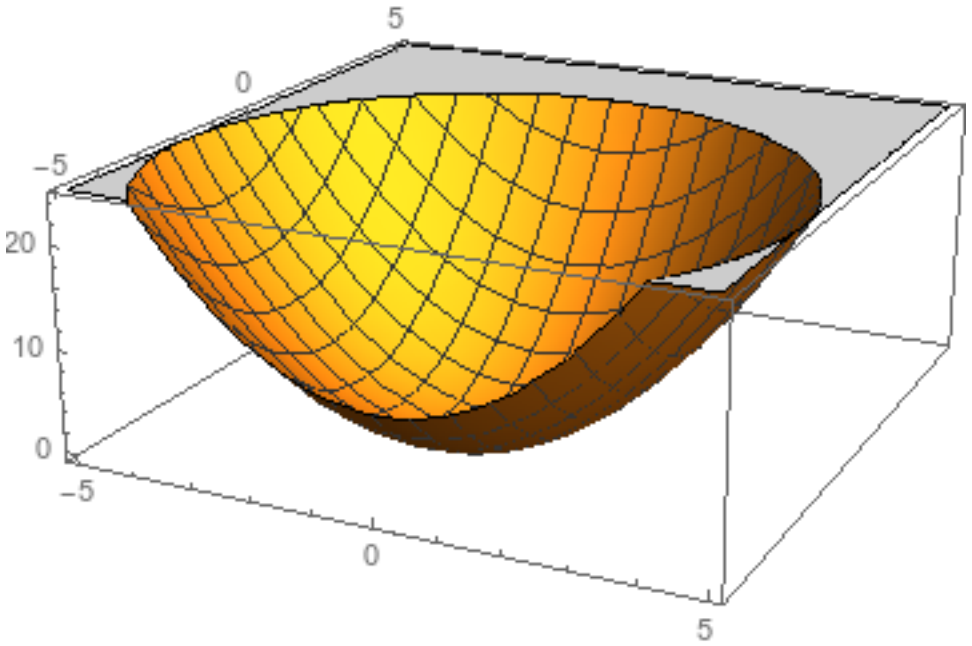

5
0
−5
10
5
0
−5
−10
−5
0
5



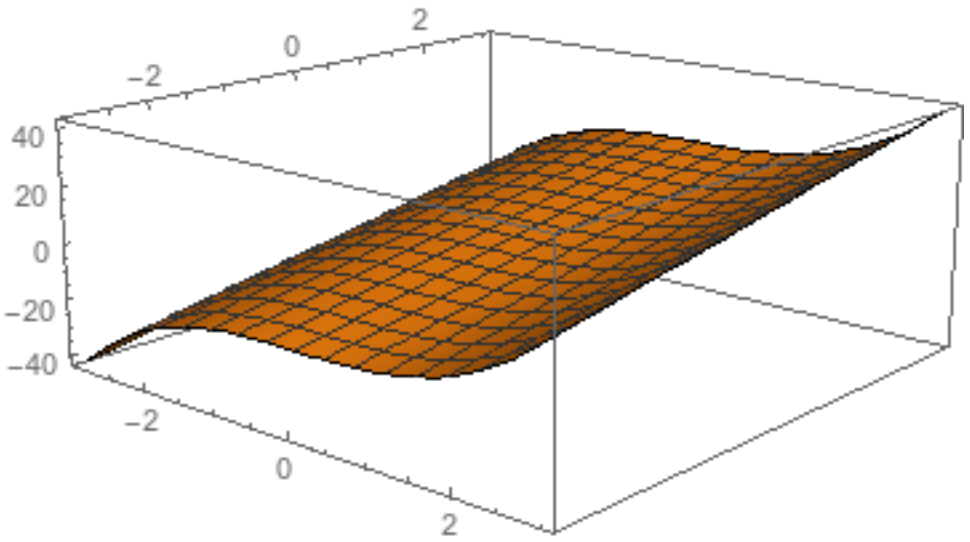
Graphs of the functions $x^2 + y^2$, $x + y$, and $\sqrt{9 - x^2 - y^2}$

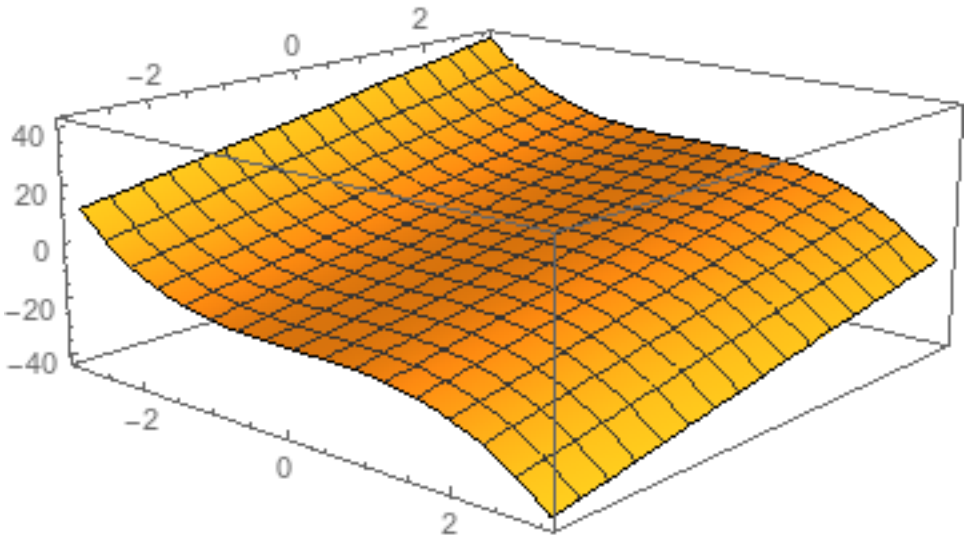

5
0
−5
20
10
0
−5
0
5

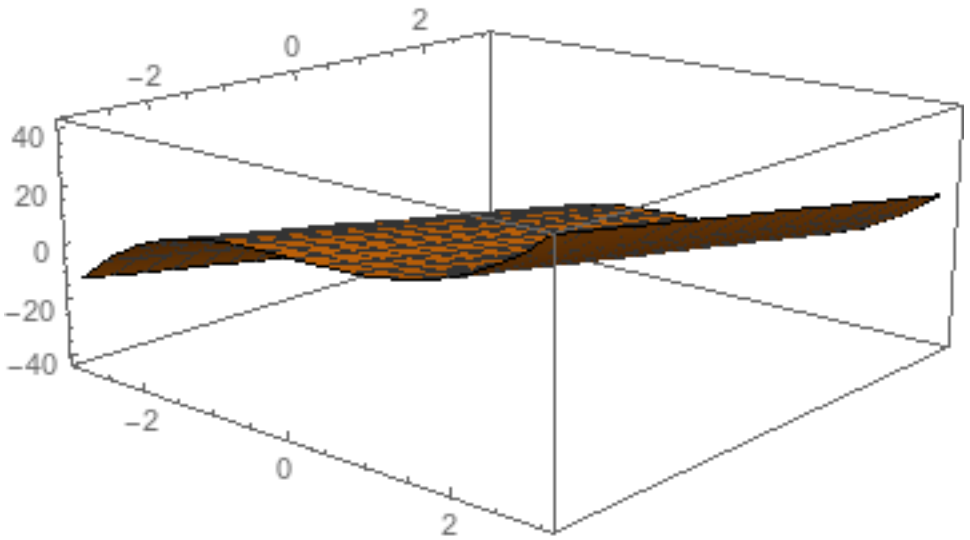

5
0
−5
20
10
0
−5
0
5

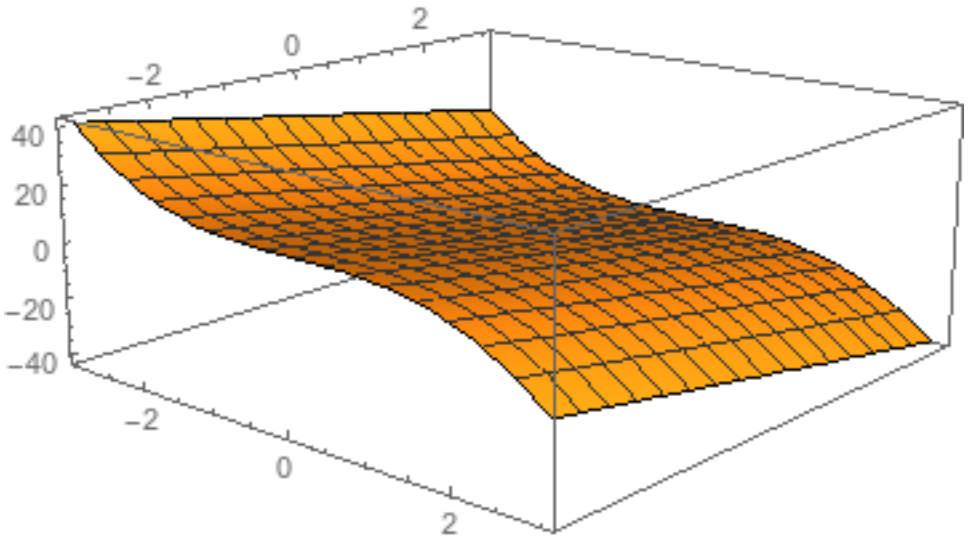


The graphs of $f(x,y)+5$, $f(x-1,y)$, and $f(x,y+2)$

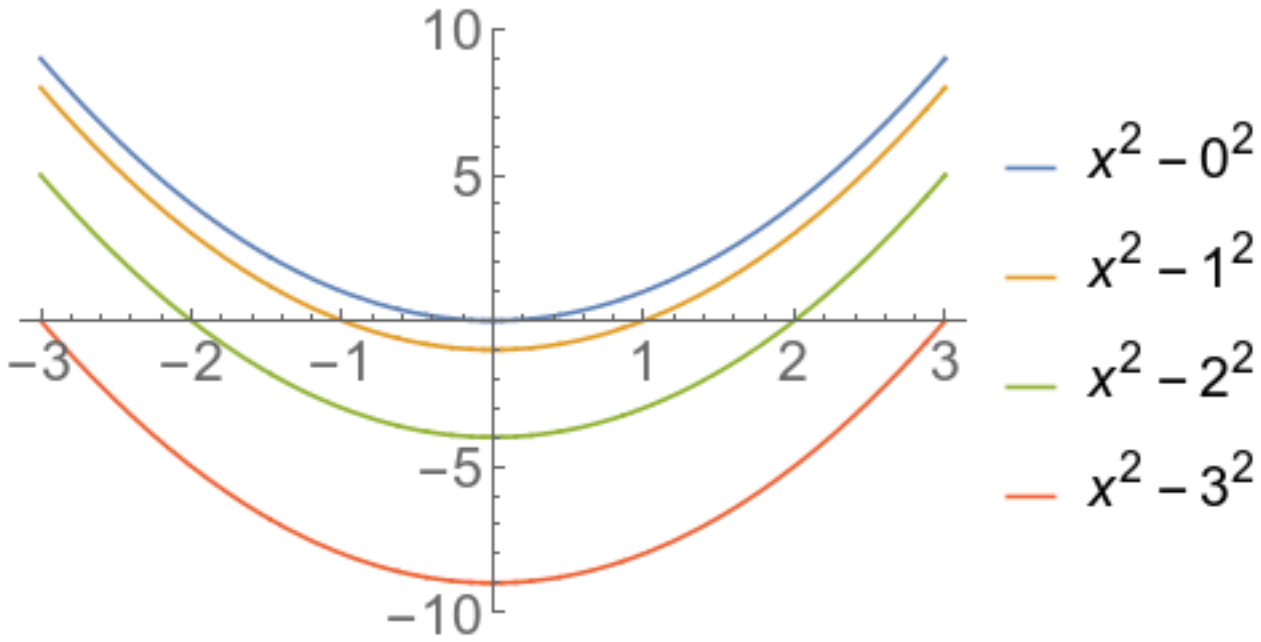

2
0
−2
40
20
0
−20
−40
−2
0
2

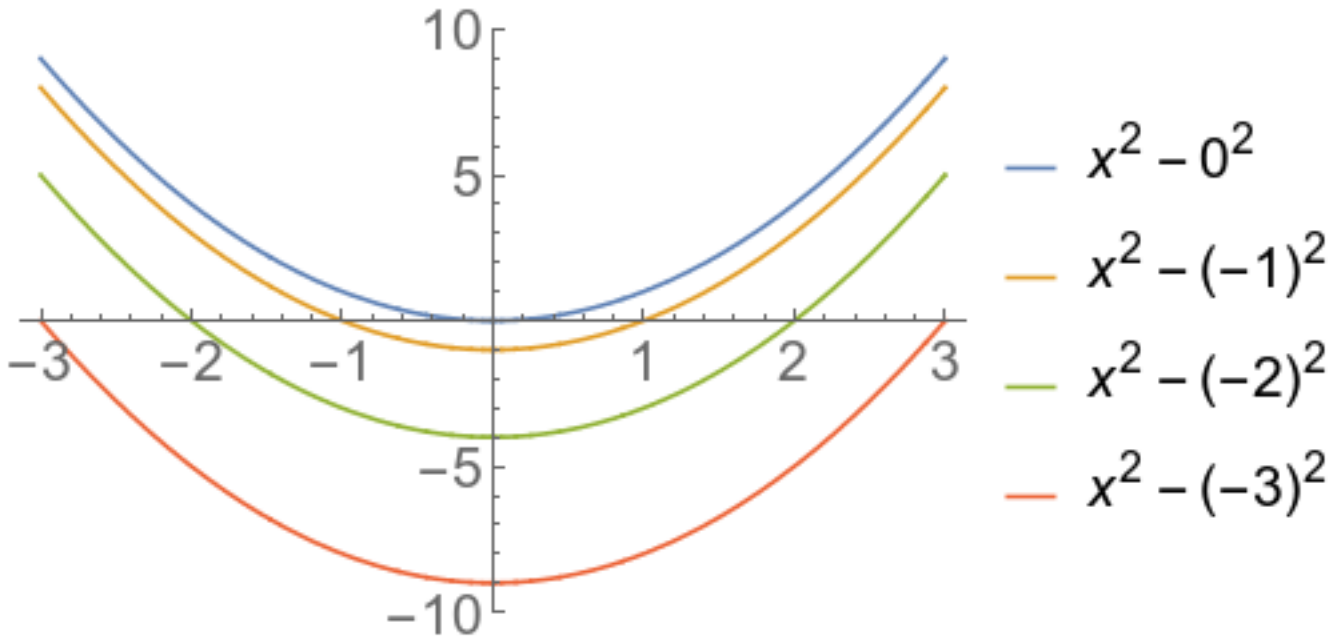

2
0
−2
40
20
0
−20
−40
−2
0
2


2
0
−2
40
20
0
−20
−40
−2
0
2

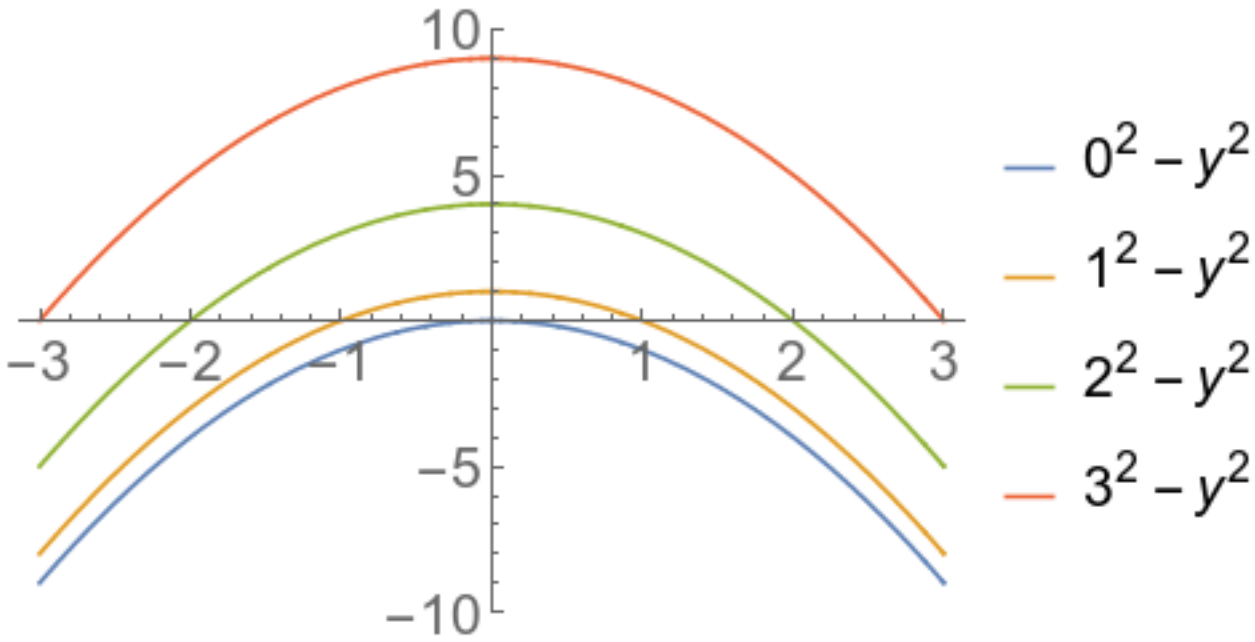


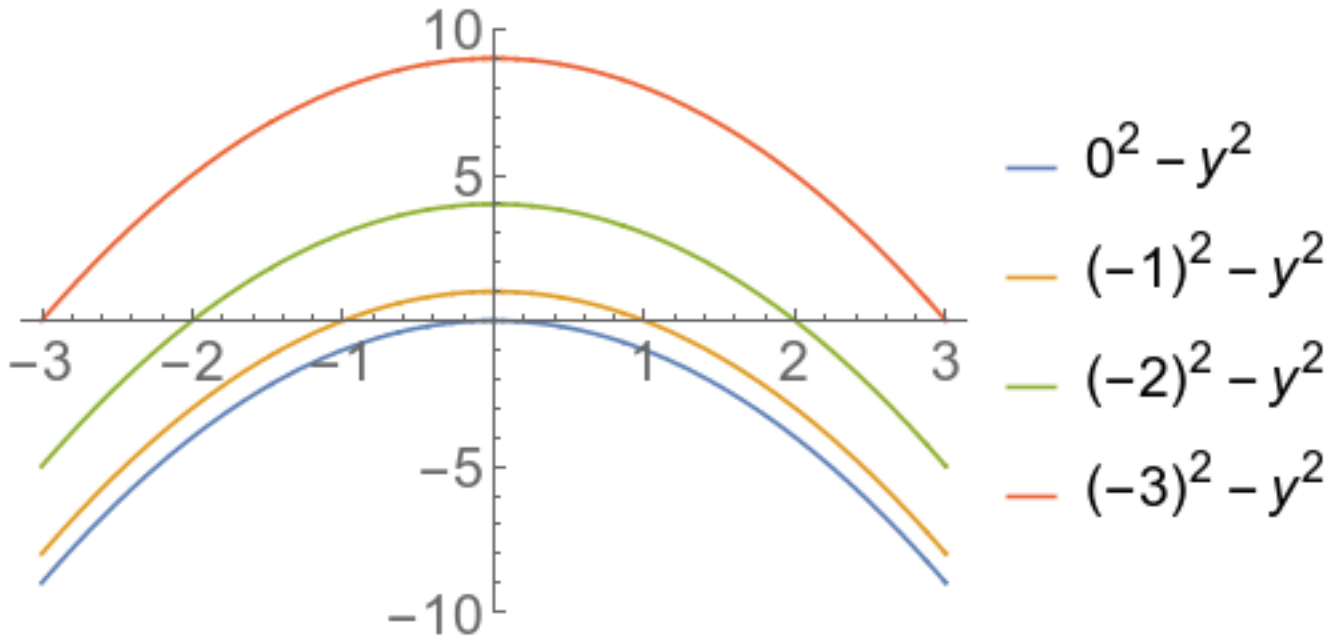
The graphs of $g(x, y)$, $g(-x, y)$, $g(x, -y)$, and $-g(x, y)$


10
5
−5
−10
−3
−2
−1
1
2
3
$x^2 - 0^2$
$x^2 - 1^2$
$x^2 - 2^2$
$x^2 - 3^2$

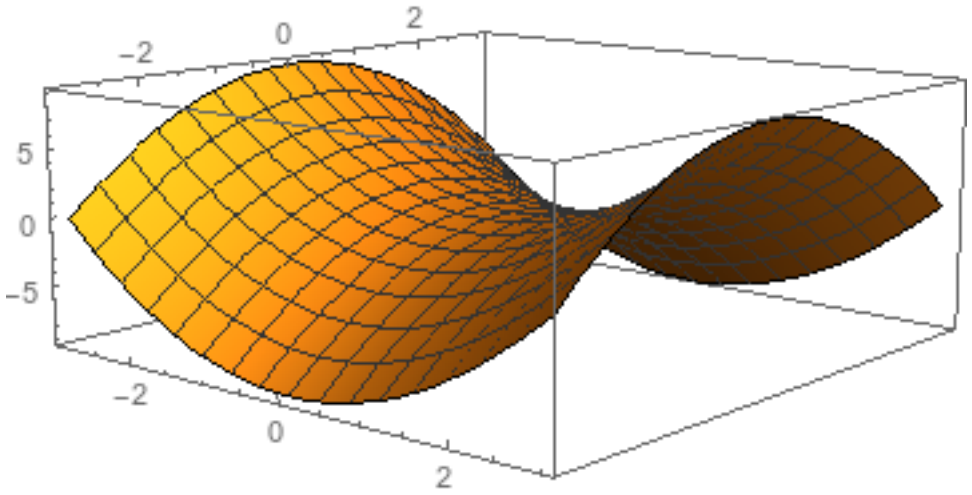


Cross sections of $x^2 - y^2$ holding y constant

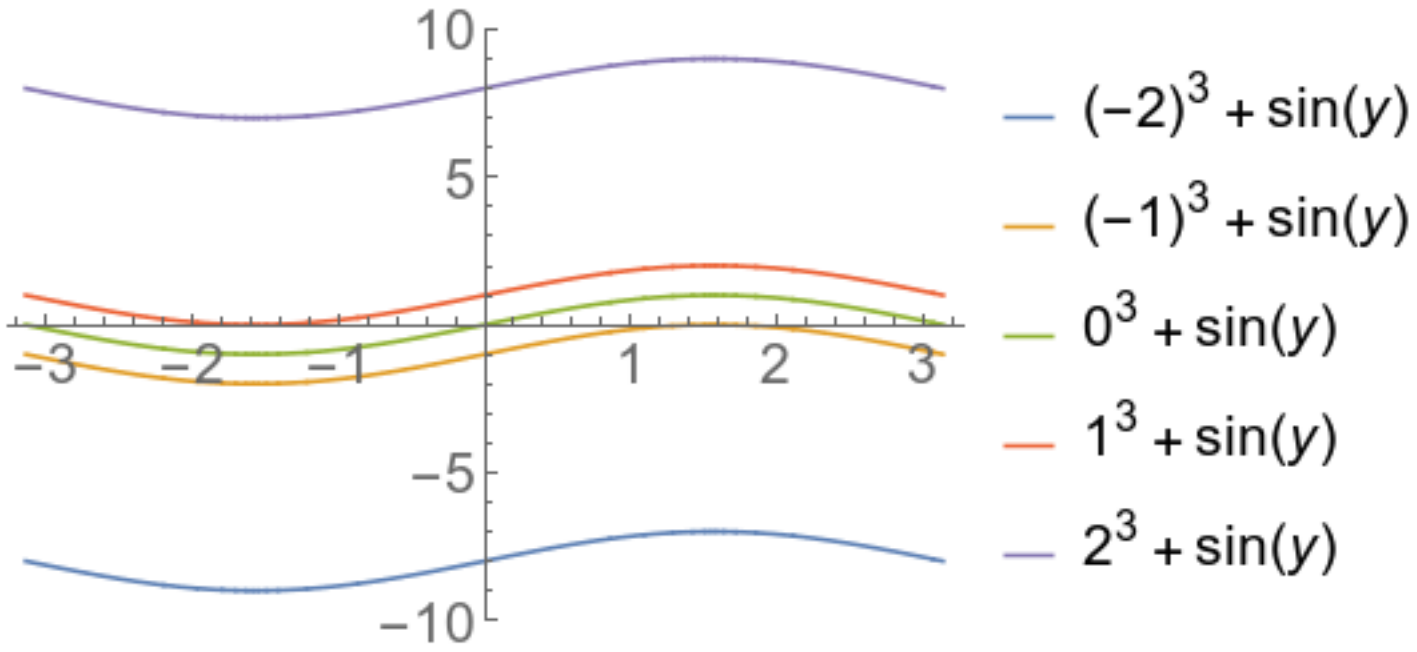

10
5
−5
−10
−3
−2
−1
1
2
3
$0^2 - y^2$
$1^2 - y^2$
$2^2 - y^2$
$3^2 - y^2$

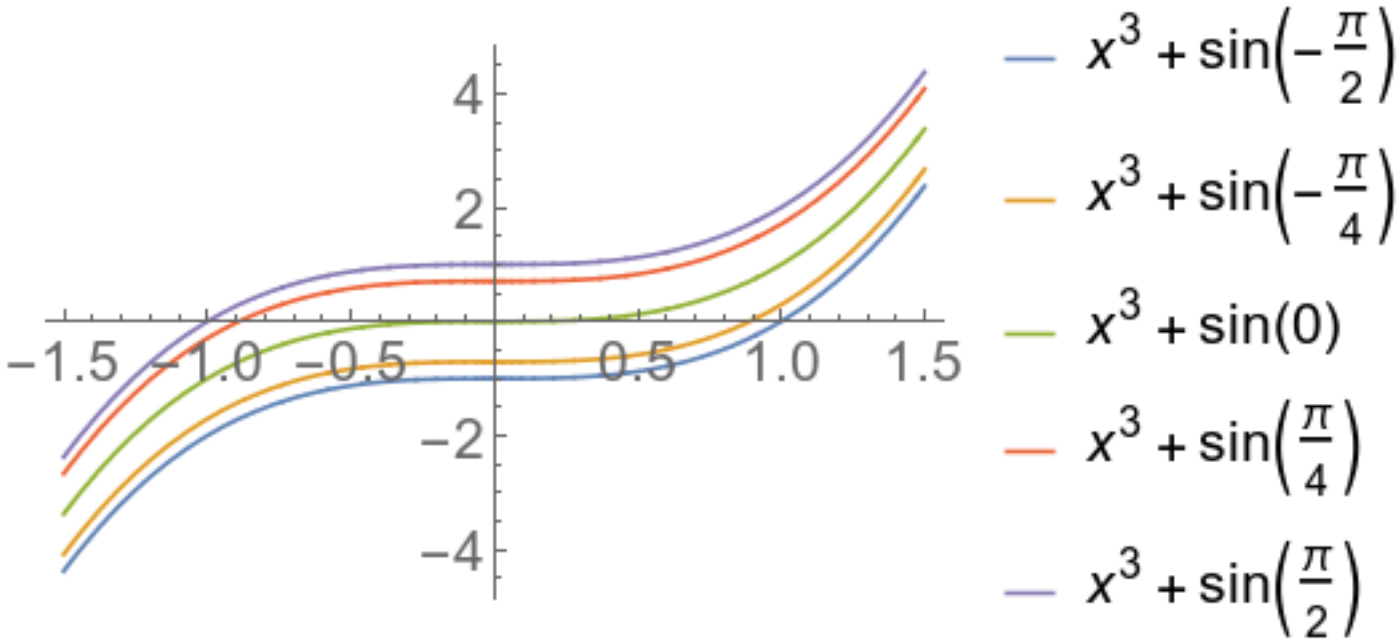


Cross sections of $x^2 - y^2$ holding x constant

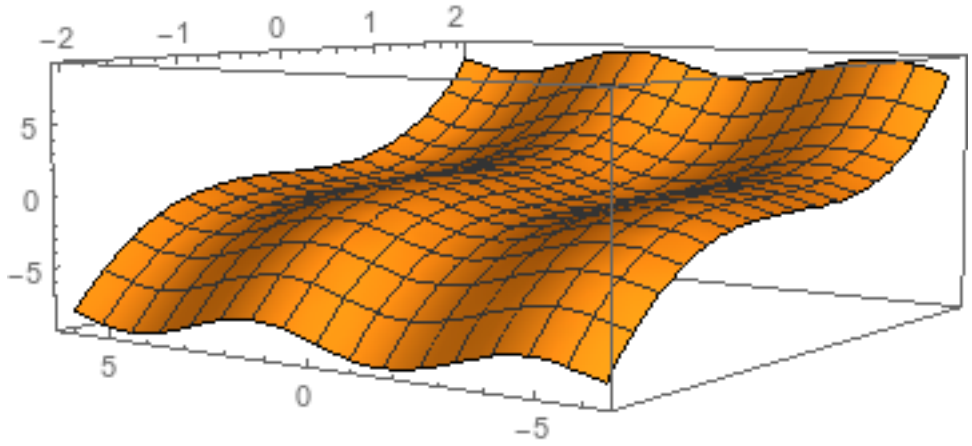


The graph of $x^2 - y^2$

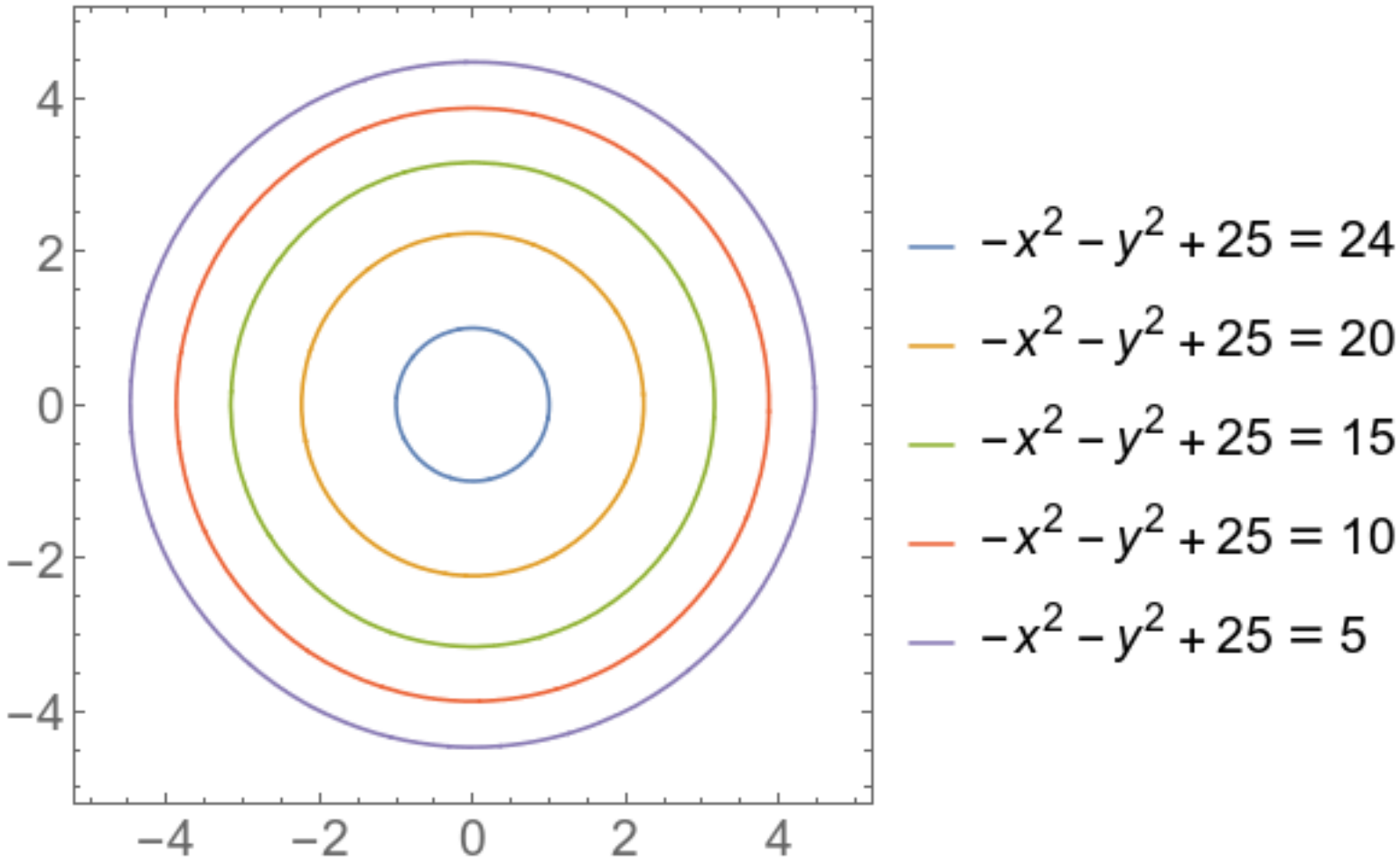

10
5
−5
−10
−3
−2
−1
1
2
3
$(-2)^3 + \sin(y)$
$(-1)^3 + \sin(y)$
$0^3 + \sin(y)$
$1^3 + \sin(y)$
$2^3 + \sin(y)$

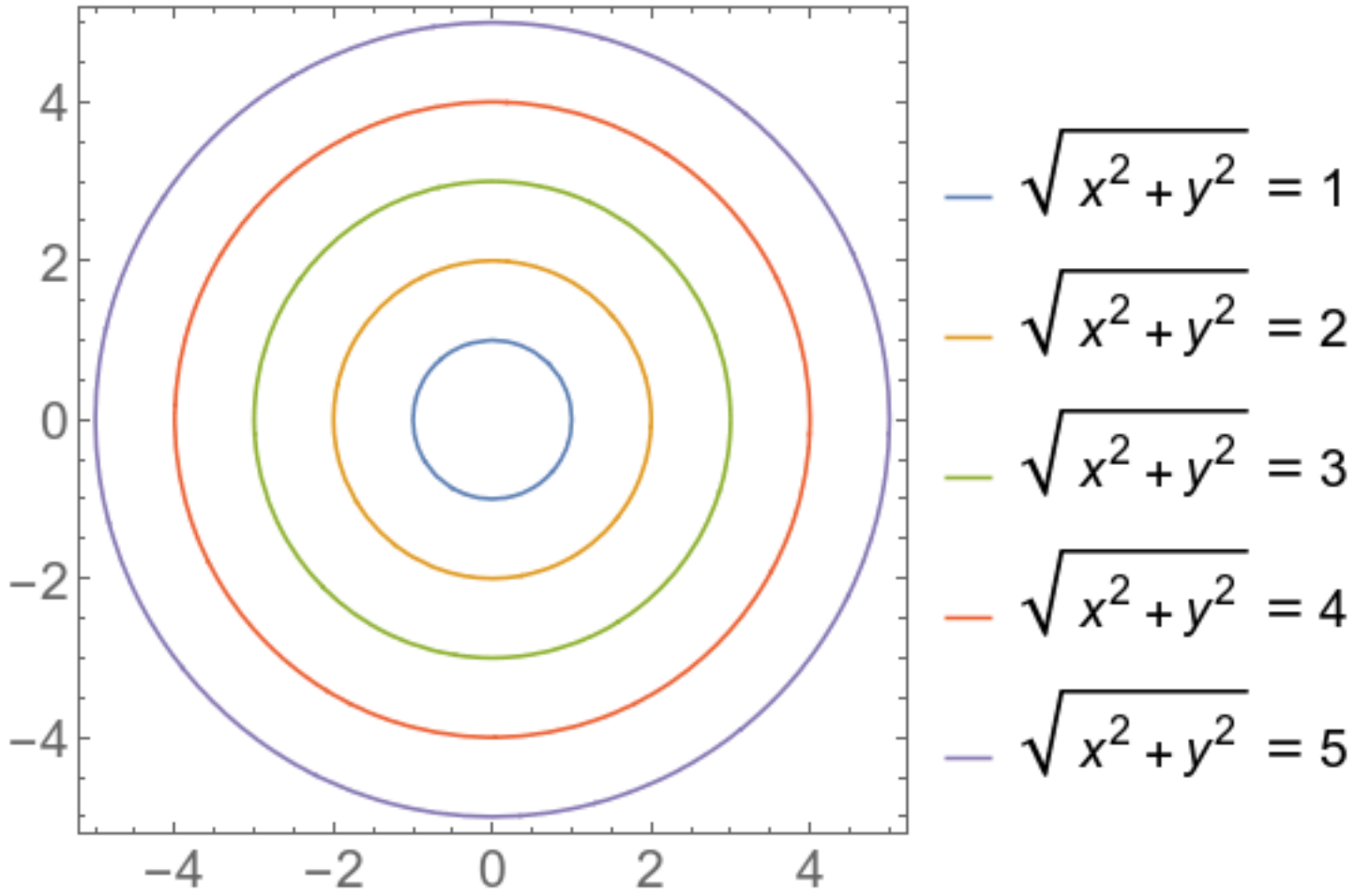


Cross sections of $x^3 + \sin(y)$, holding x constant on the left and y constant on the right



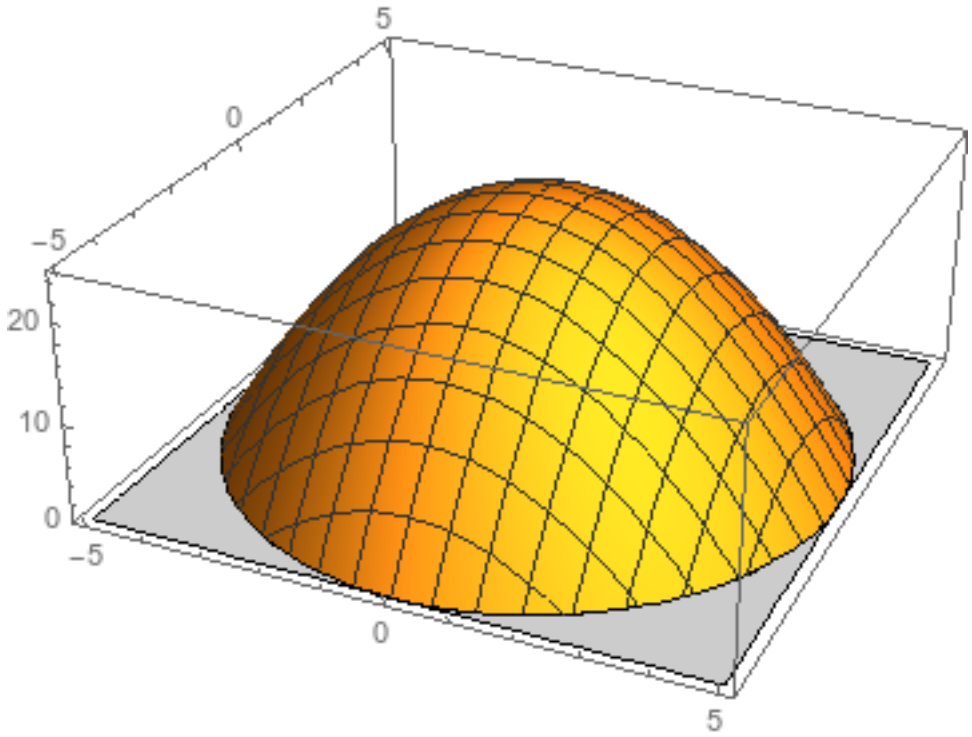
The graph of $x^3 + \sin(y)$

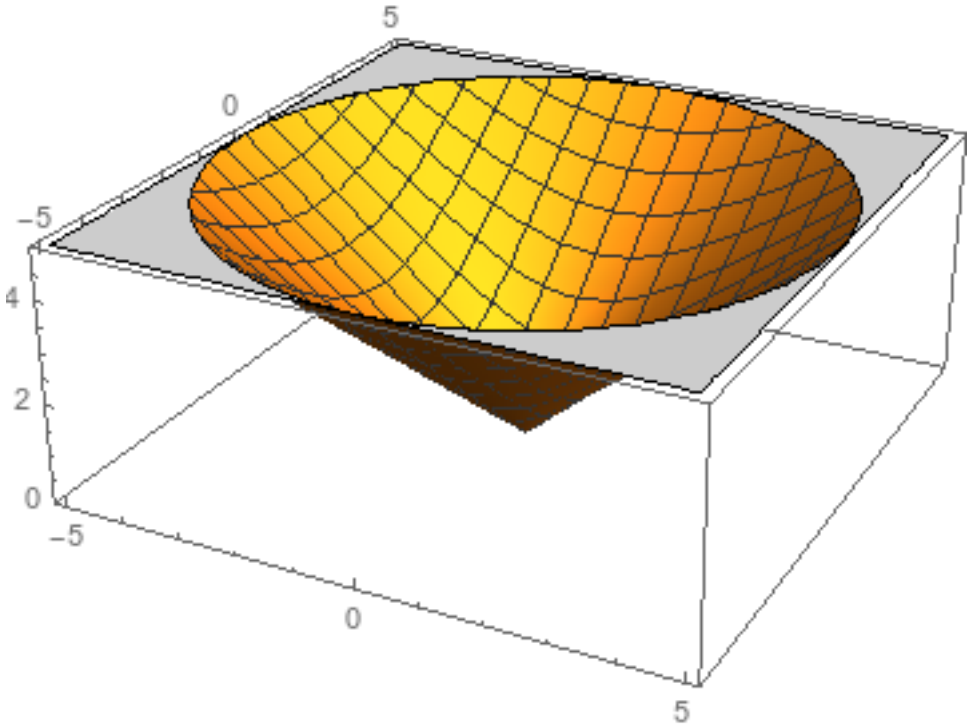

4
2
0
−2
−4
−4
−2
0
2
4
$-x^2-y^2+25=24$
$-x^2-y^2+25=20$
$-x^2-y^2+25=15$
$-x^2-y^2+25=10$
$-x^2-y^2+25=5$



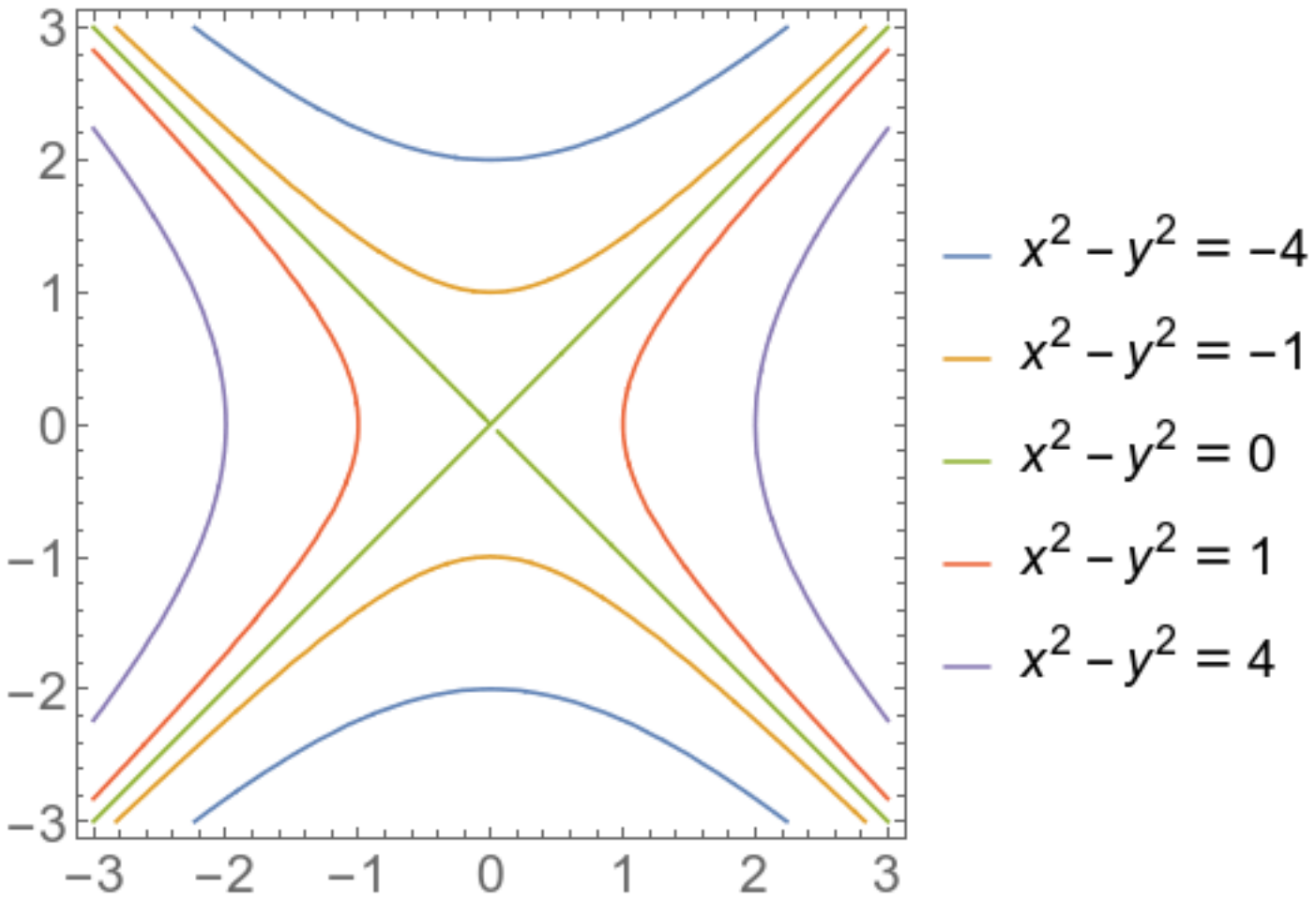
Contour diagrams

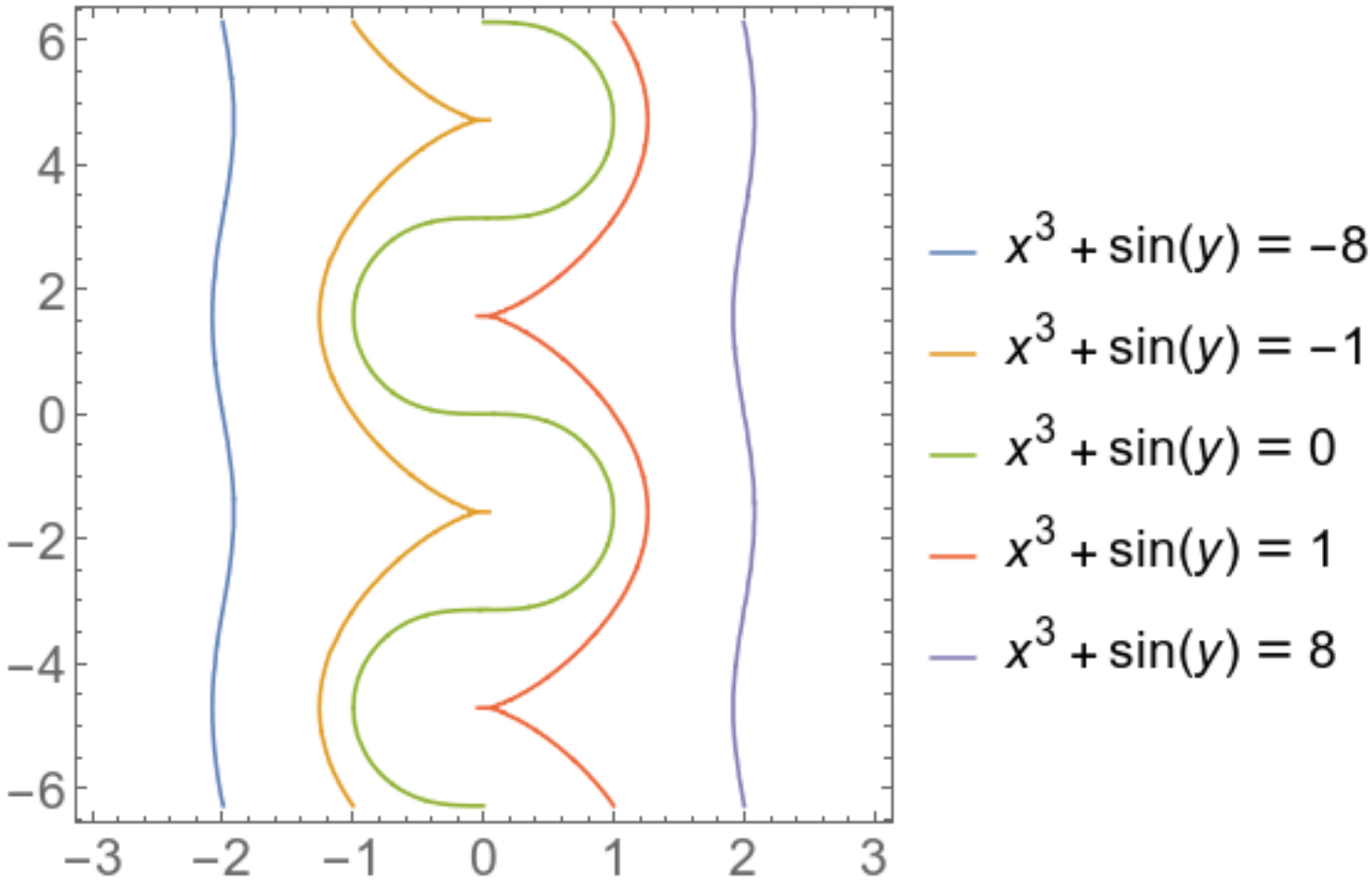
for $f(x,y) = 25 - x^2 - y^2$ and $g(x,y) = \sqrt{x^2+y^2}$


5
0
−5
20
10
0
−5
0
5



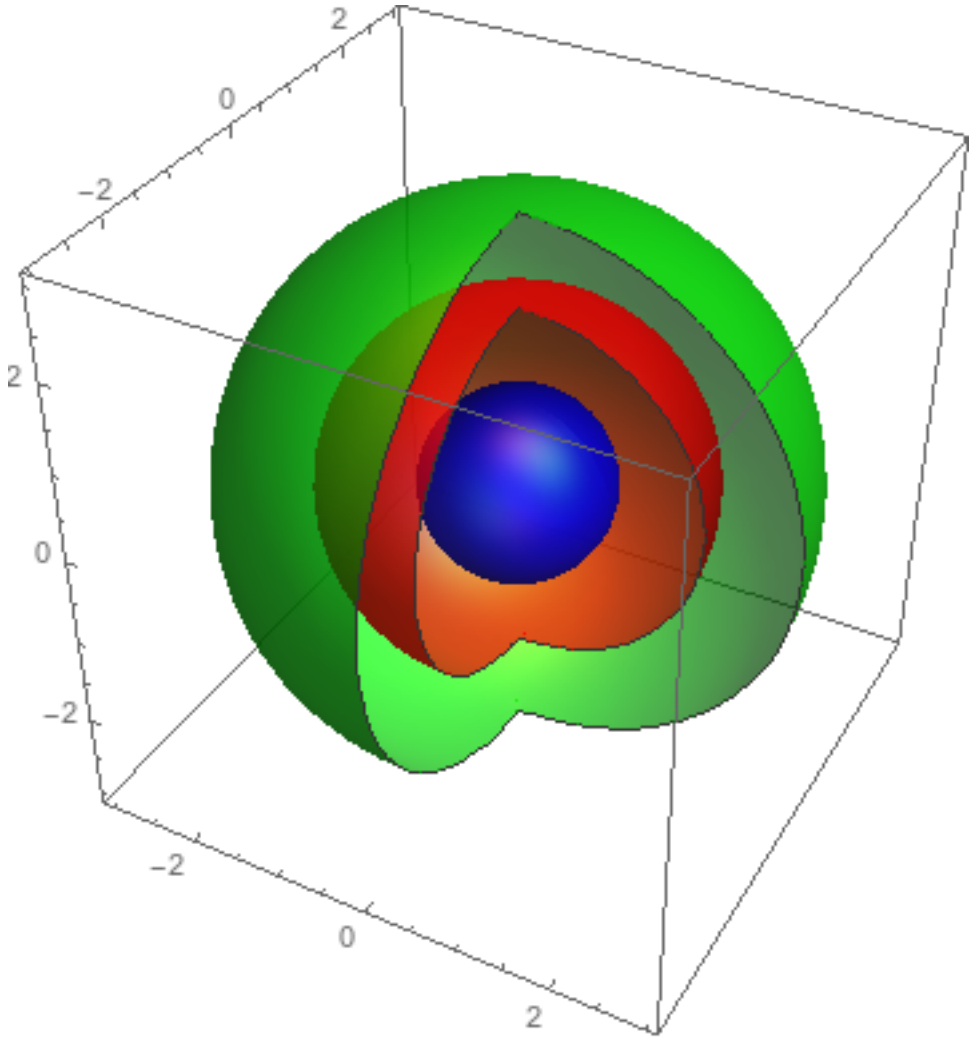
The graphs of $f(x,y) = 25 - x^2 - y^2$ and $g(x,y) = \sqrt{x^2+y^2}$


3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
$x^2 - y^2 = -4$
$x^2 - y^2 = -1$
$x^2 - y^2 = 0$
$x^2 - y^2 = 1$
$x^2 - y^2 = 4$

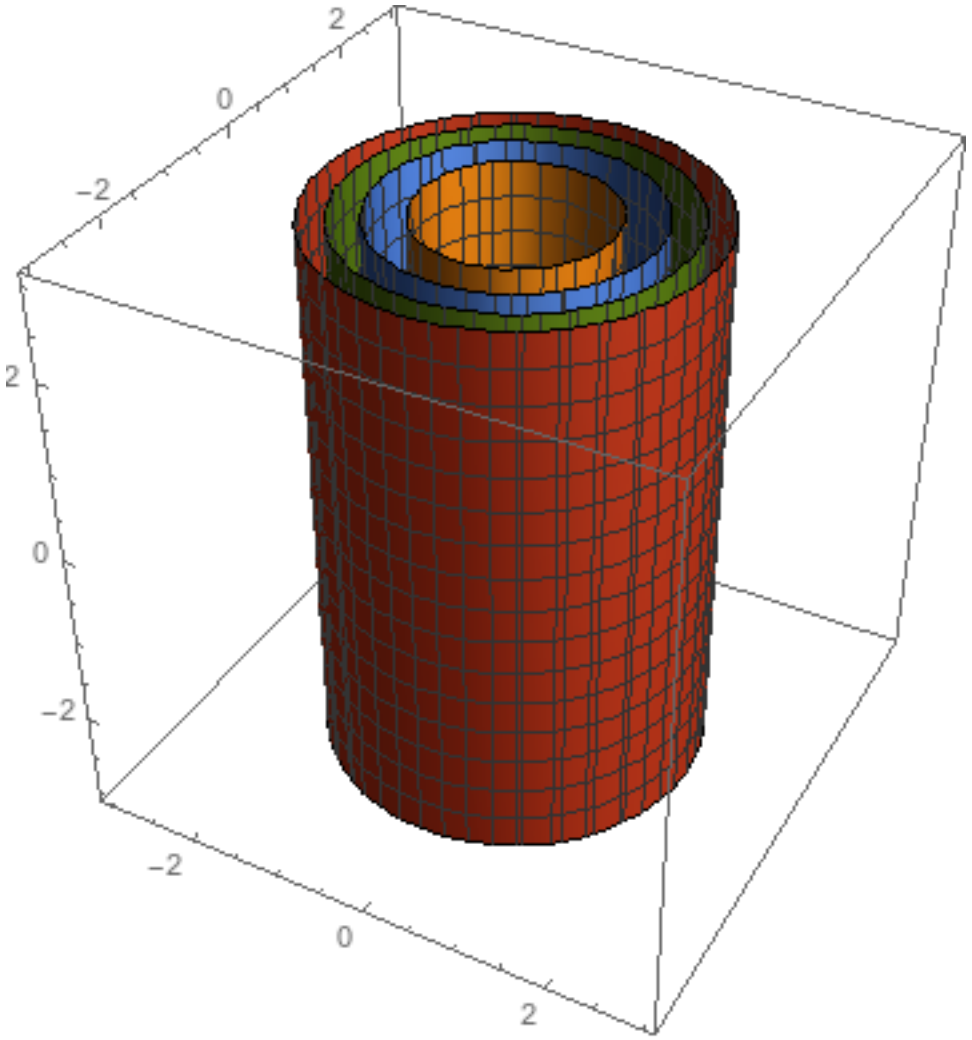


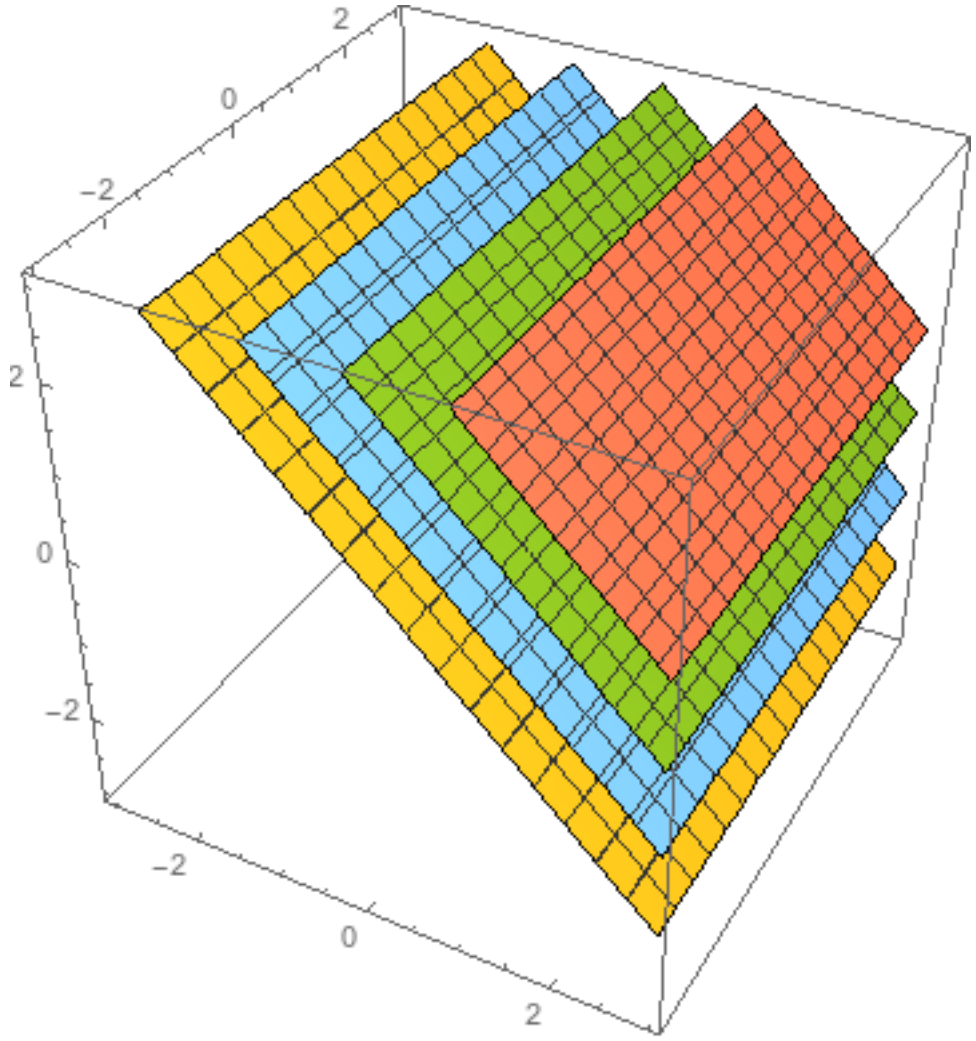
for $x^2 - y^2$ and $x^3 + \sin(y)$

Contour plots

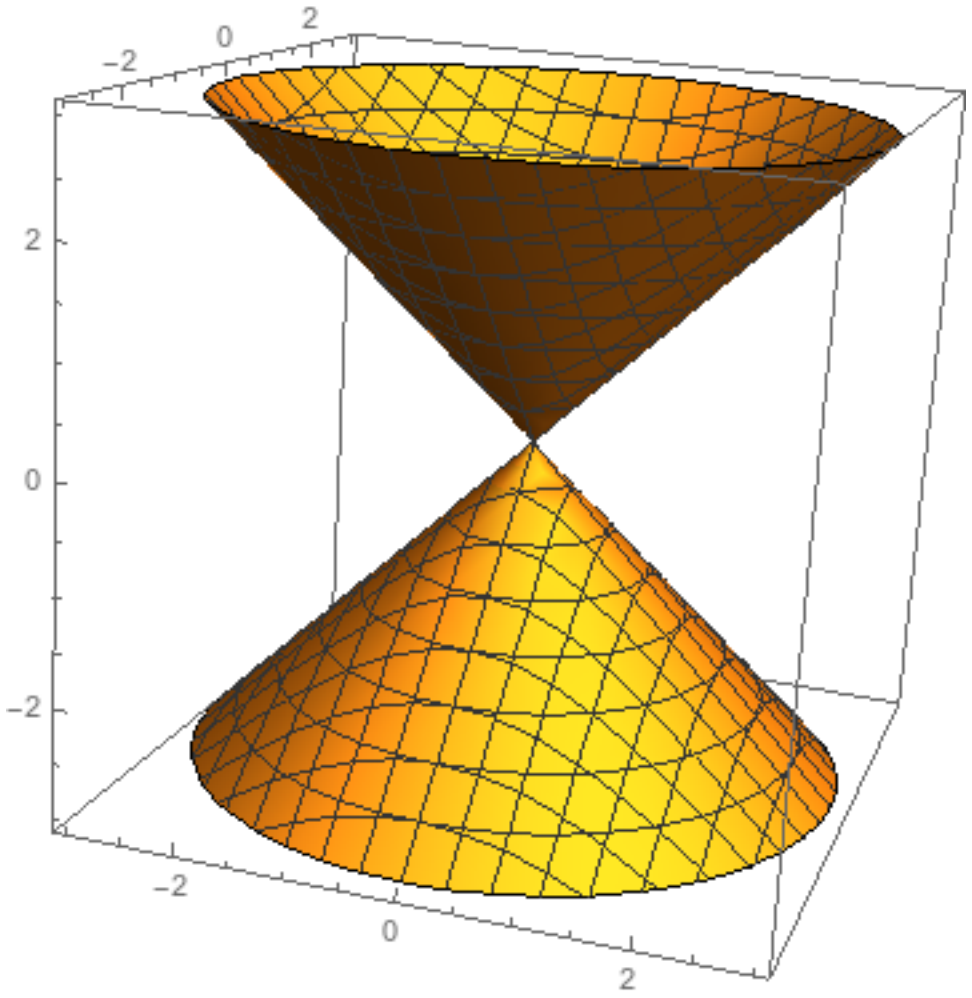


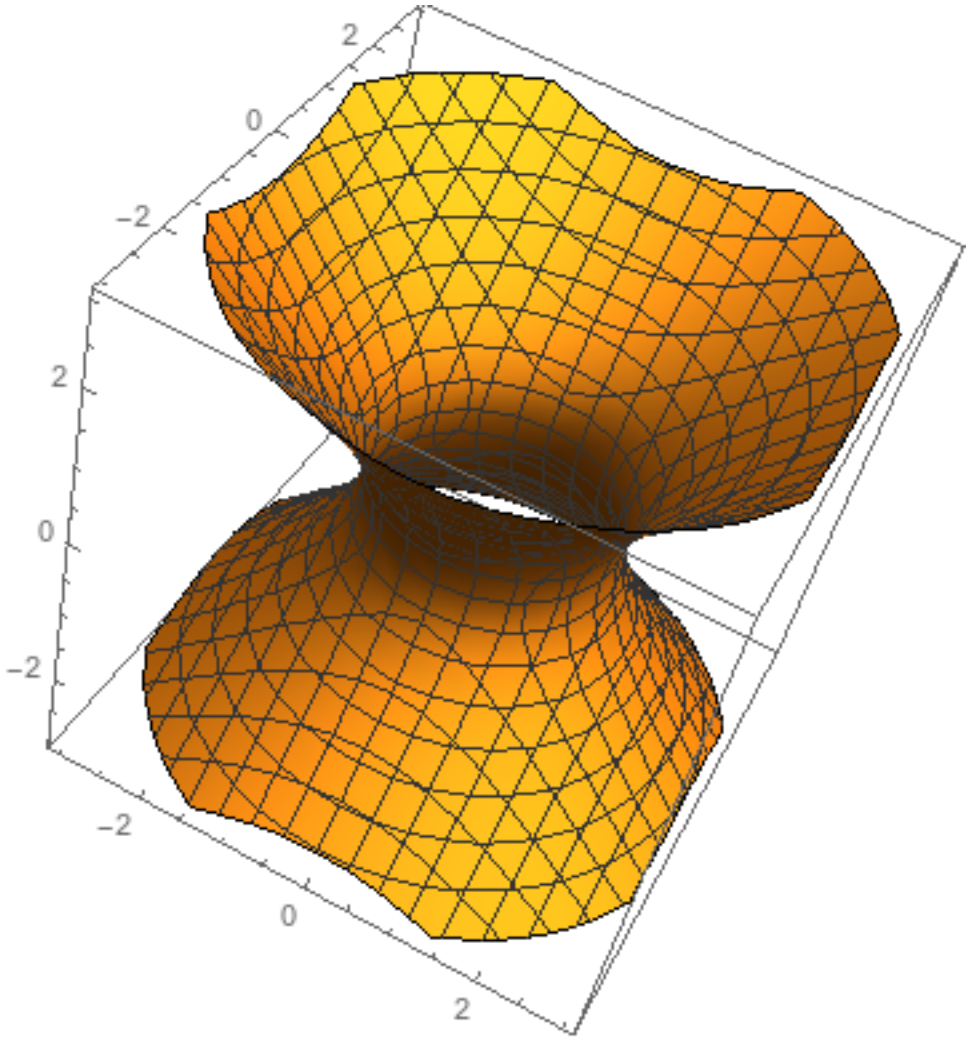
Level sets for $x^2 + y^2 + z^2$ at the levels $c = 1, 4, 9$

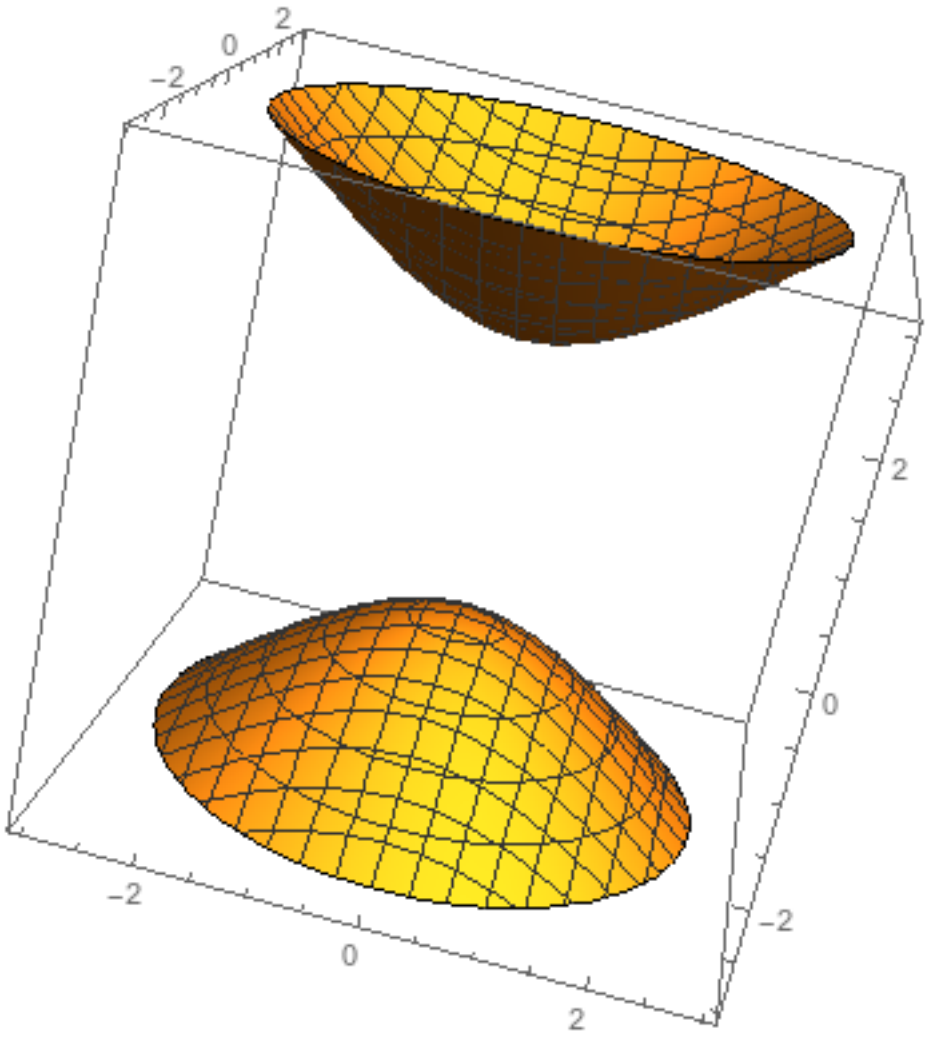

2
0
−2
2
0
−2
−2
0
2



The level surfaces of g and h at the levels $1, 2, 3, 4$

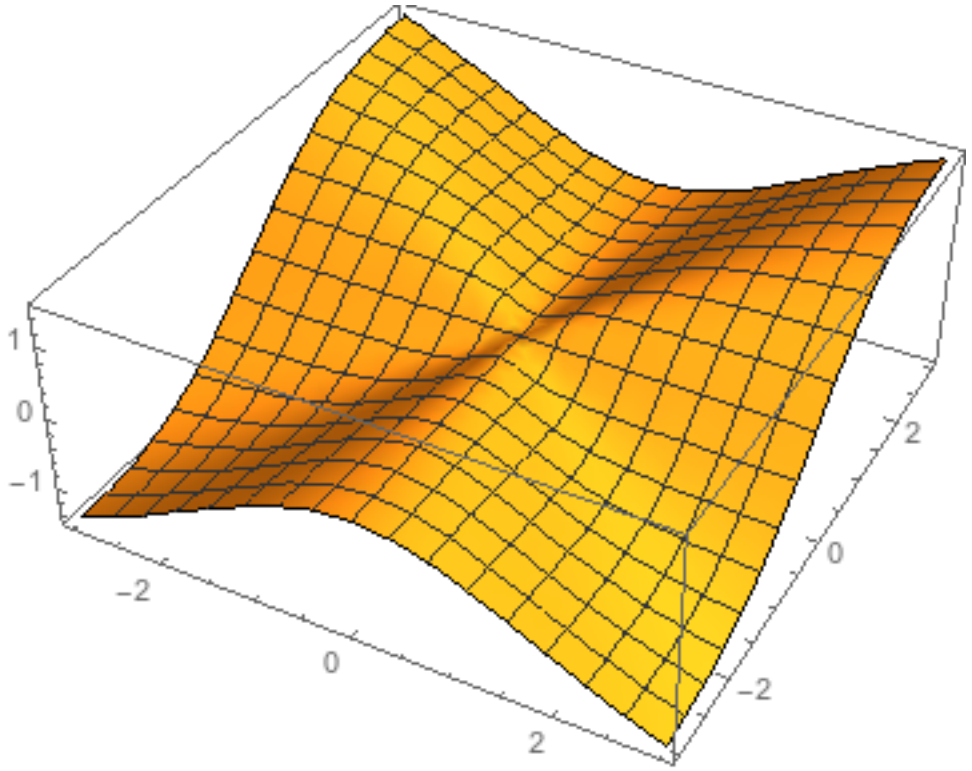

2
0
−2
2
0
−2
−2
0
2

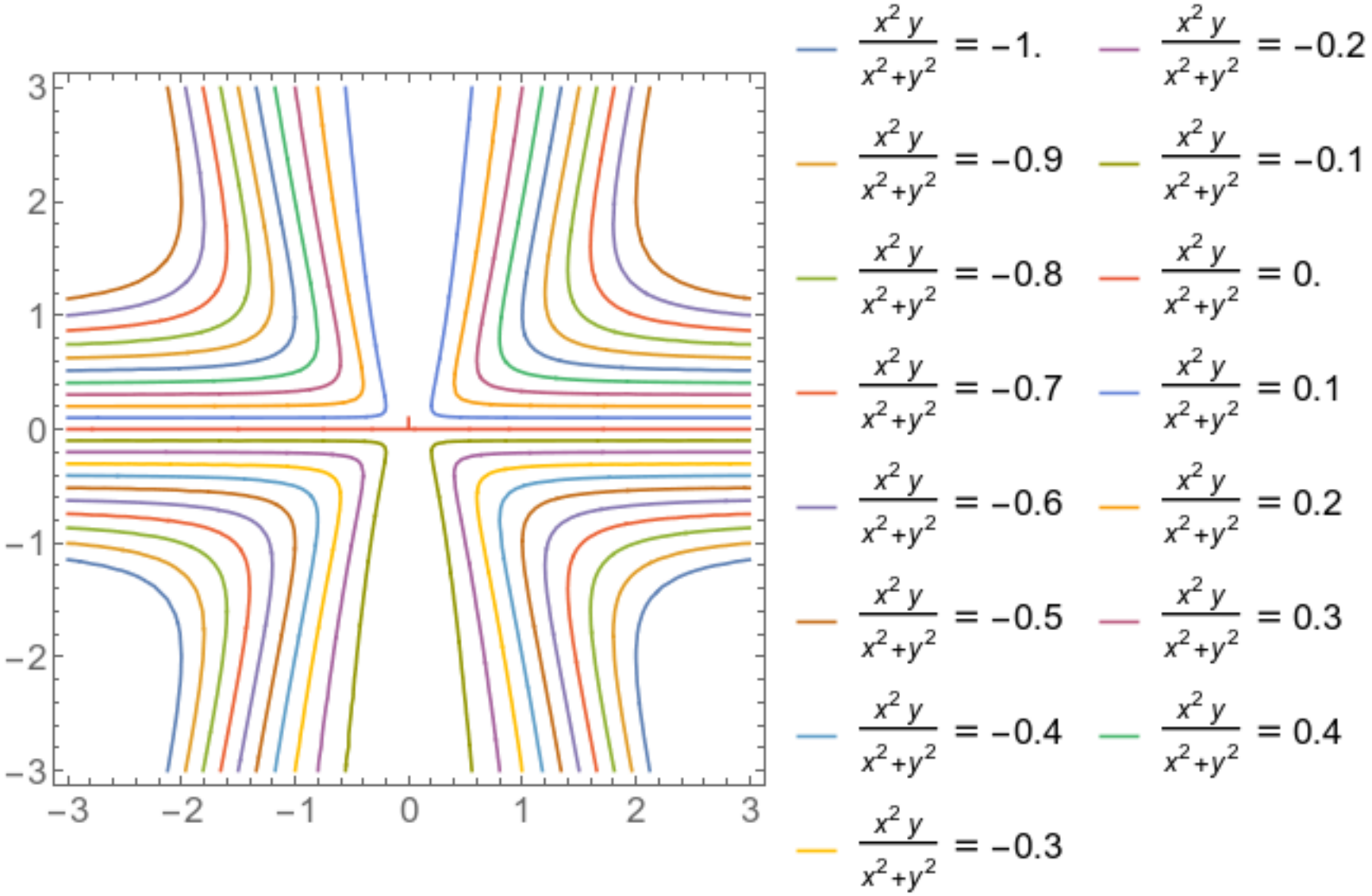

2
0
−2
2
0
−2
−2
0
2



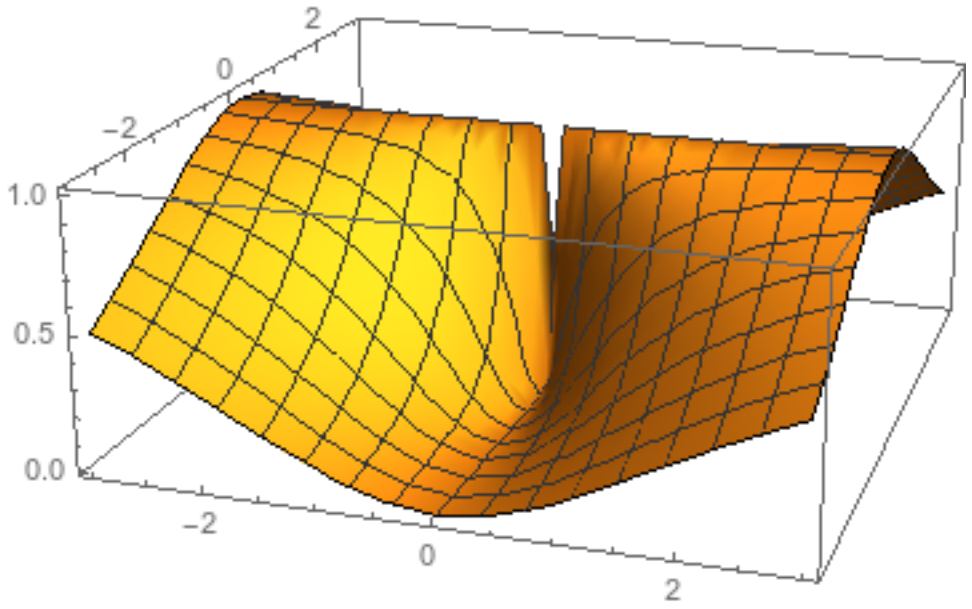
Level surfaces of $x^2 + y^2 - z^2$ at the levels $0, 2, -2$

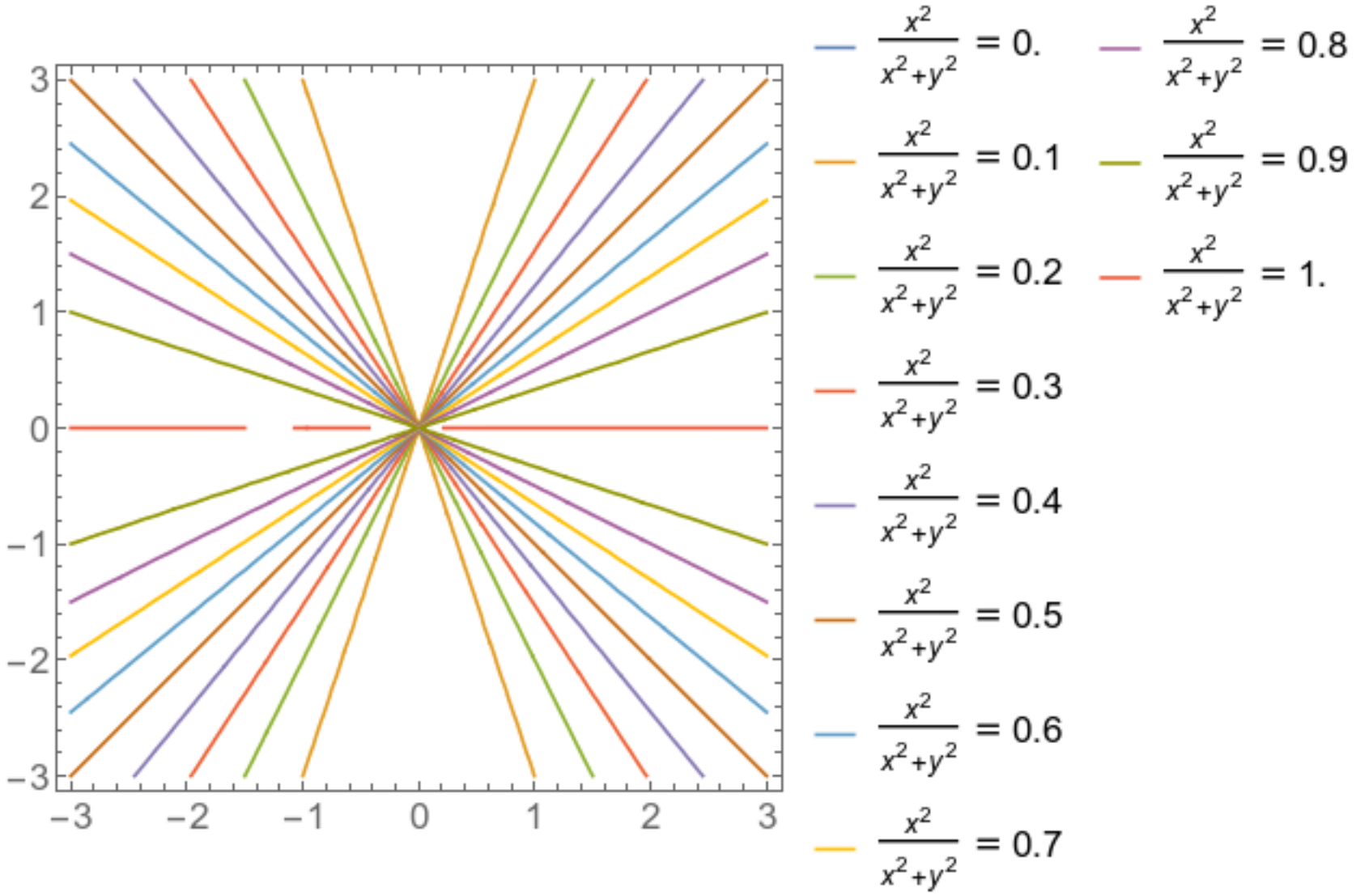
3.2 Limits and Continuity





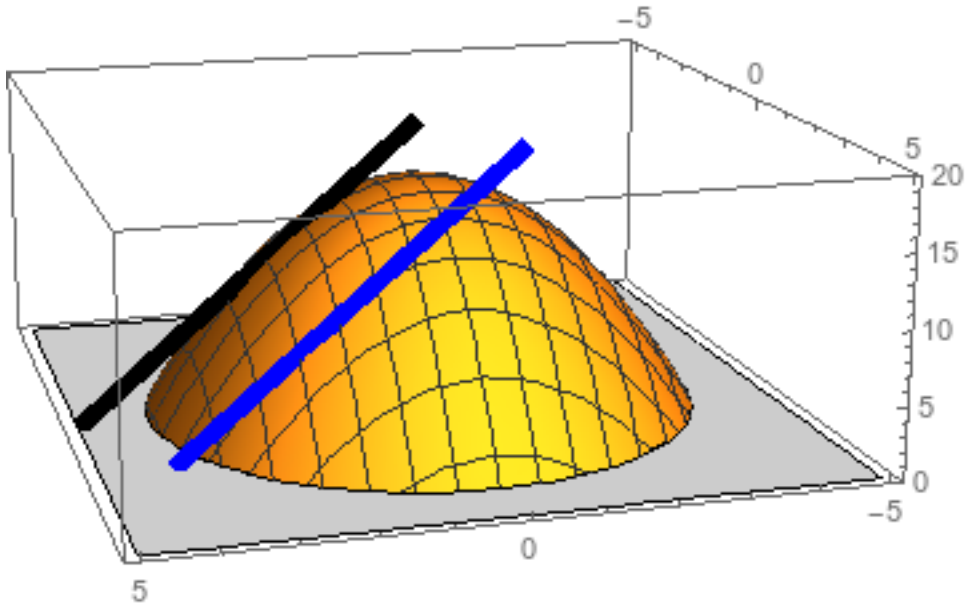
The graph and contour plot for $f(x,y) = \frac{x^2y}{x^2+y^2}$

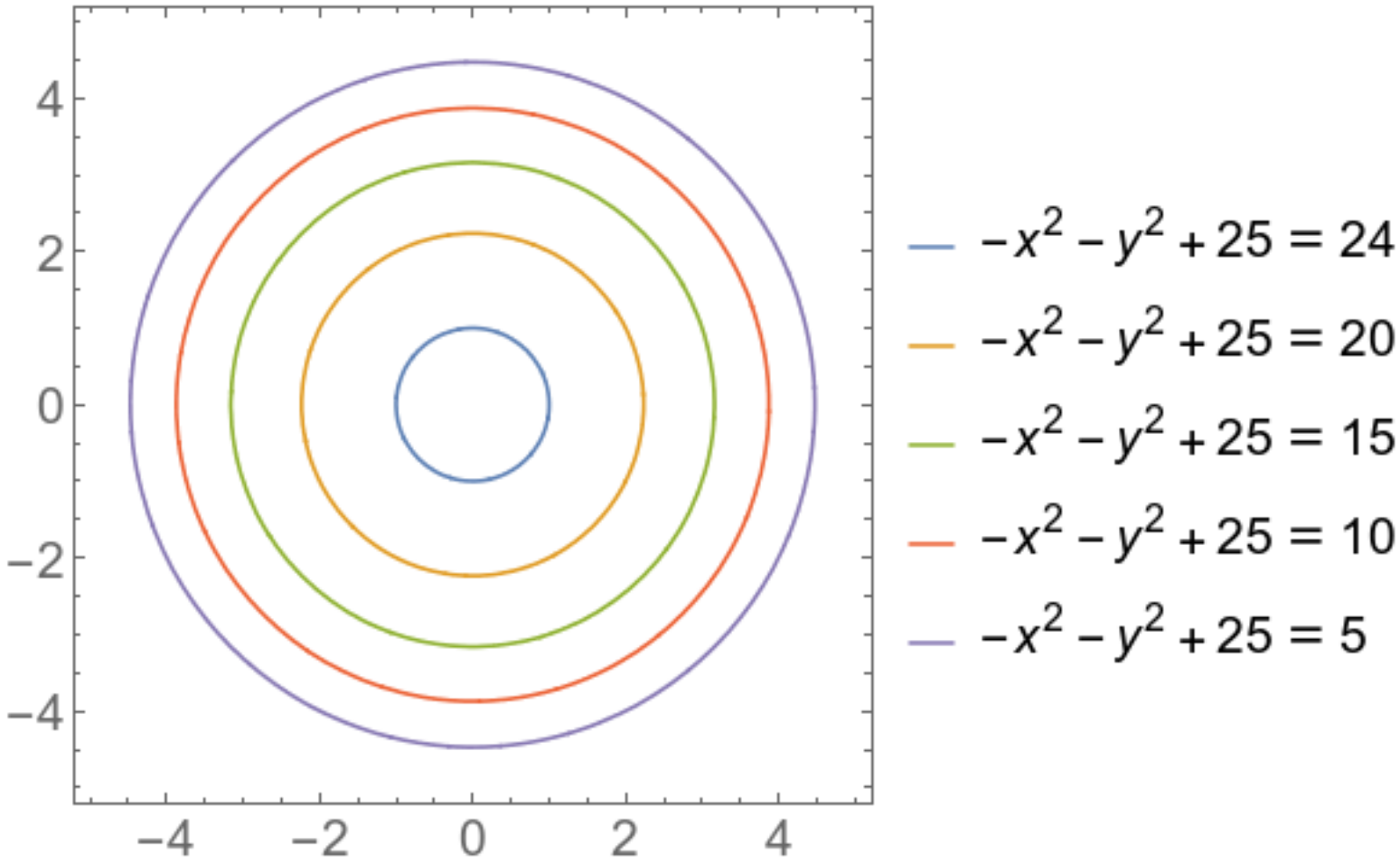

2
0
−2
1.0
0.5
0.0
−2
0
2

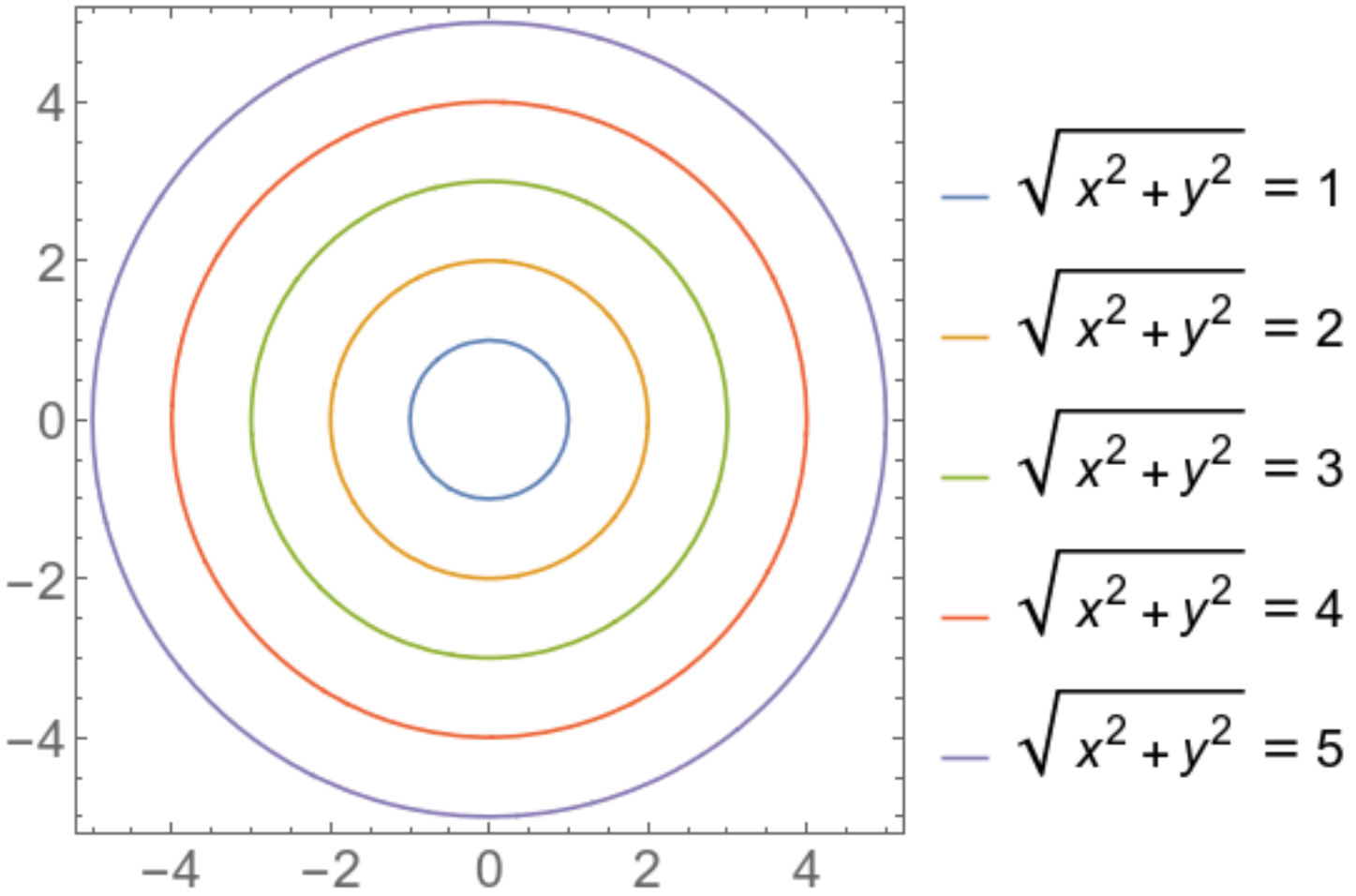


The graph and contour plot for $g(x, y) = \frac{x^2}{x^2+y^2}$

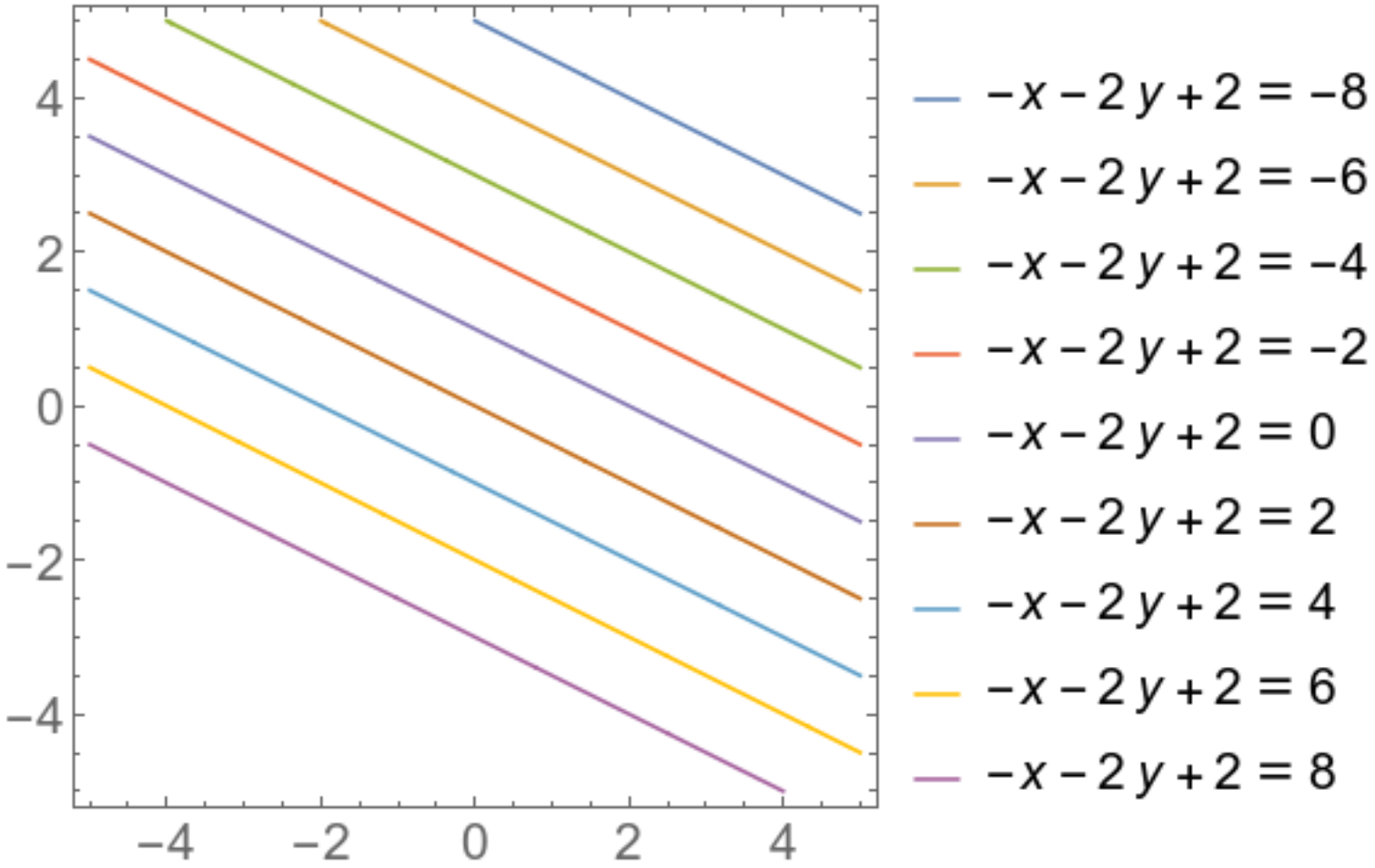
3.3 The Partial Derivative

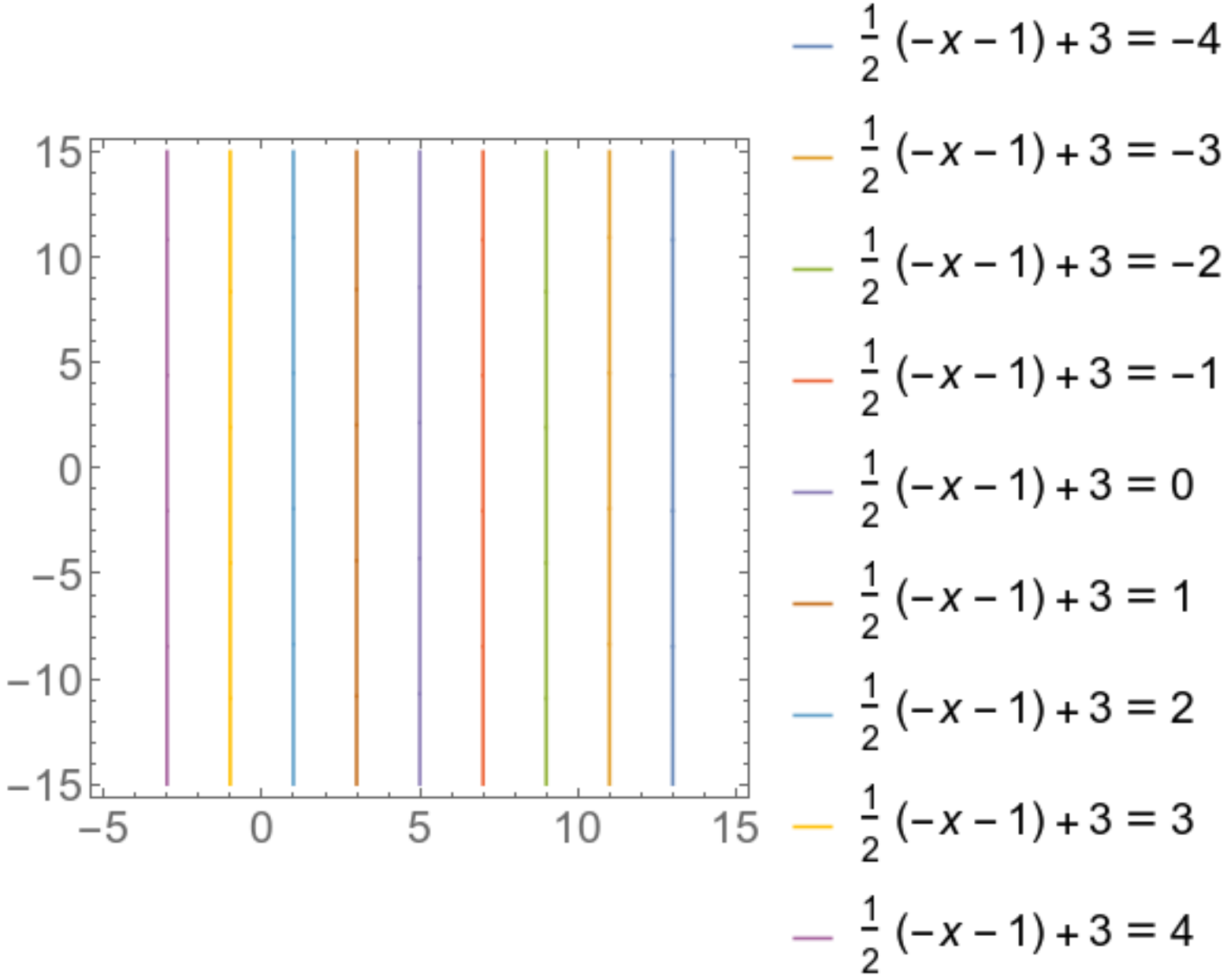


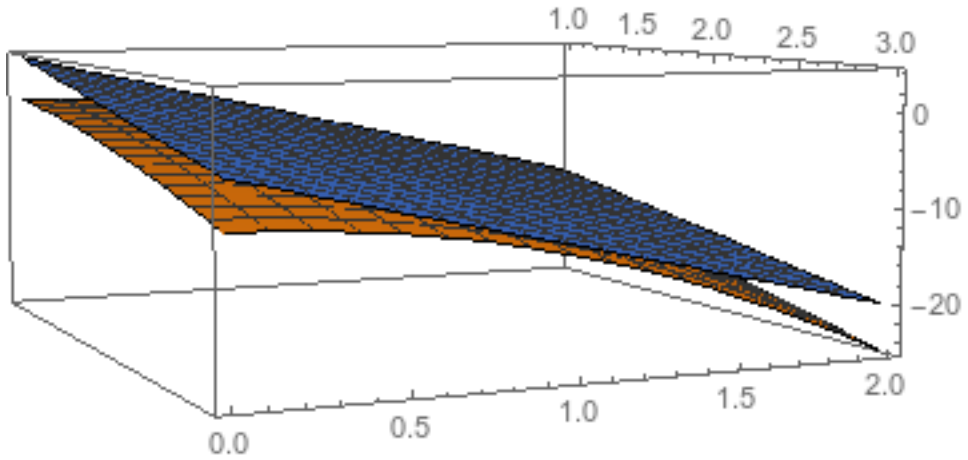

4
2
0
−2
−4
−4
−2
0
2
4
$-x^2-y^2+25=24$
$-x^2-y^2+25=20$
$-x^2-y^2+25=15$
$-x^2-y^2+25=10$
$-x^2-y^2+25=5$

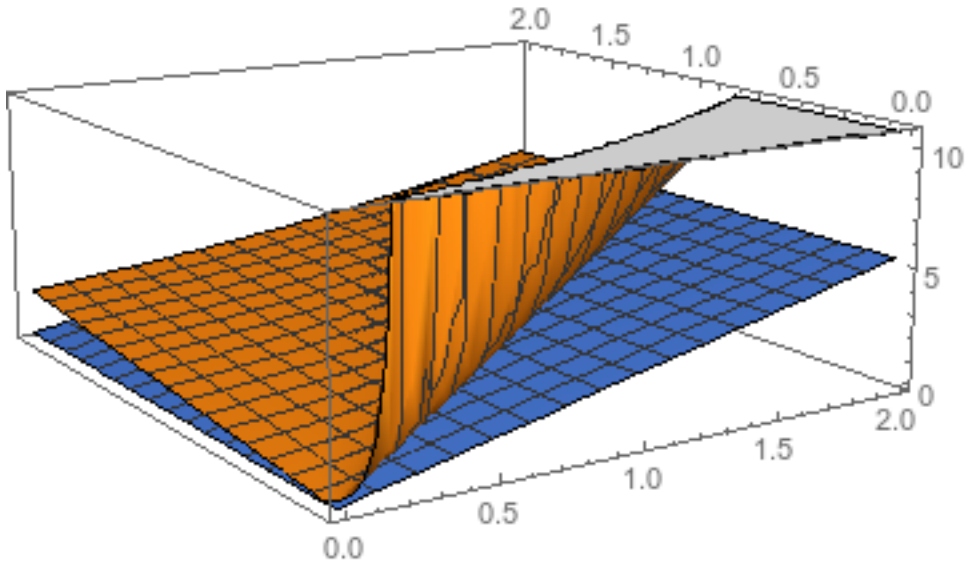

4
2
0
−2
−4
−4
−2
0
2
4
$\sqrt{x^2+y^2} = 1$
$\sqrt{x^2+y^2} = 2$
$\sqrt{x^2+y^2} = 3$
$\sqrt{x^2+y^2} = 4$
$\sqrt{x^2+y^2} = 5$

3.4 Local Linear Approximation

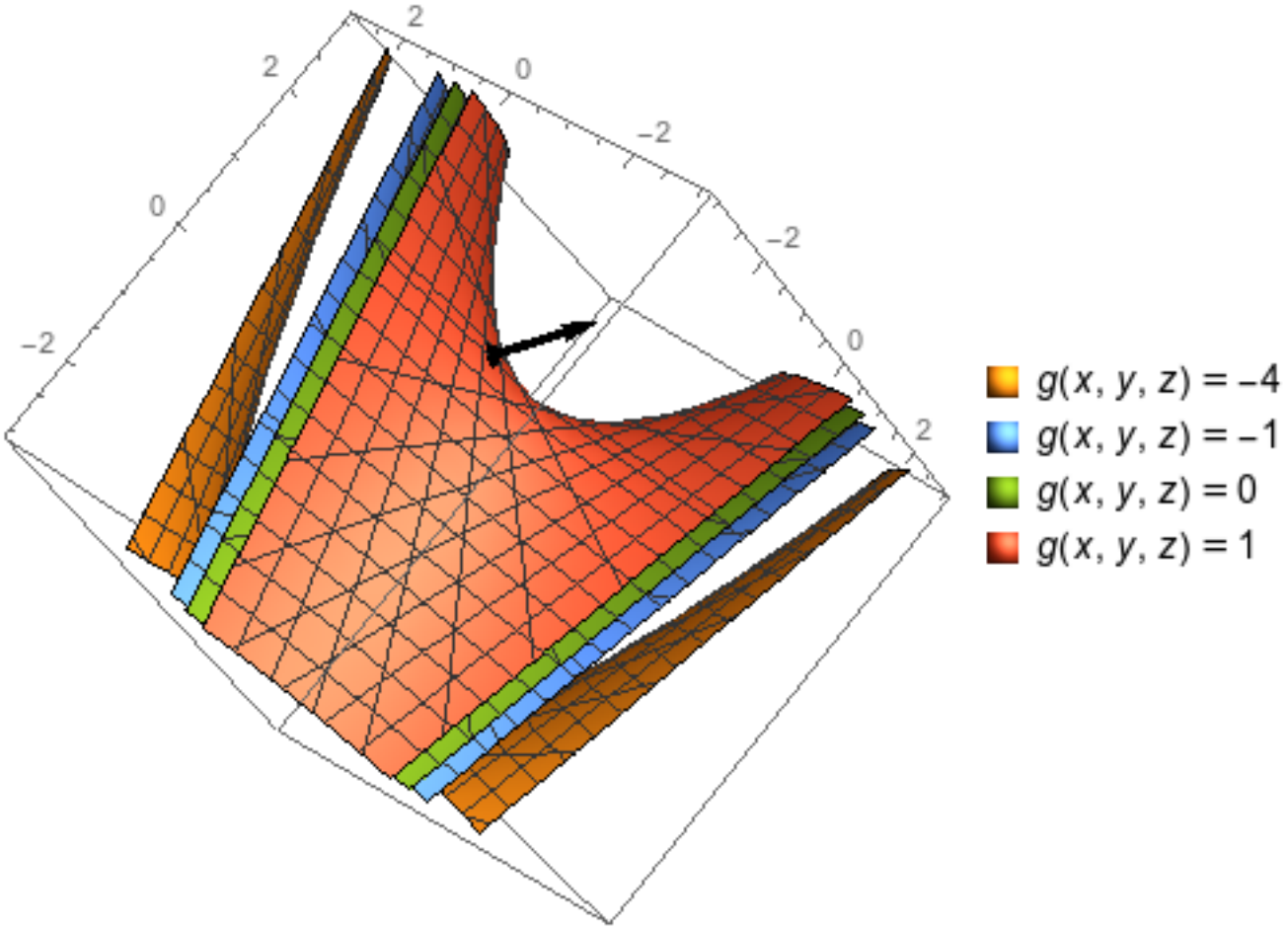


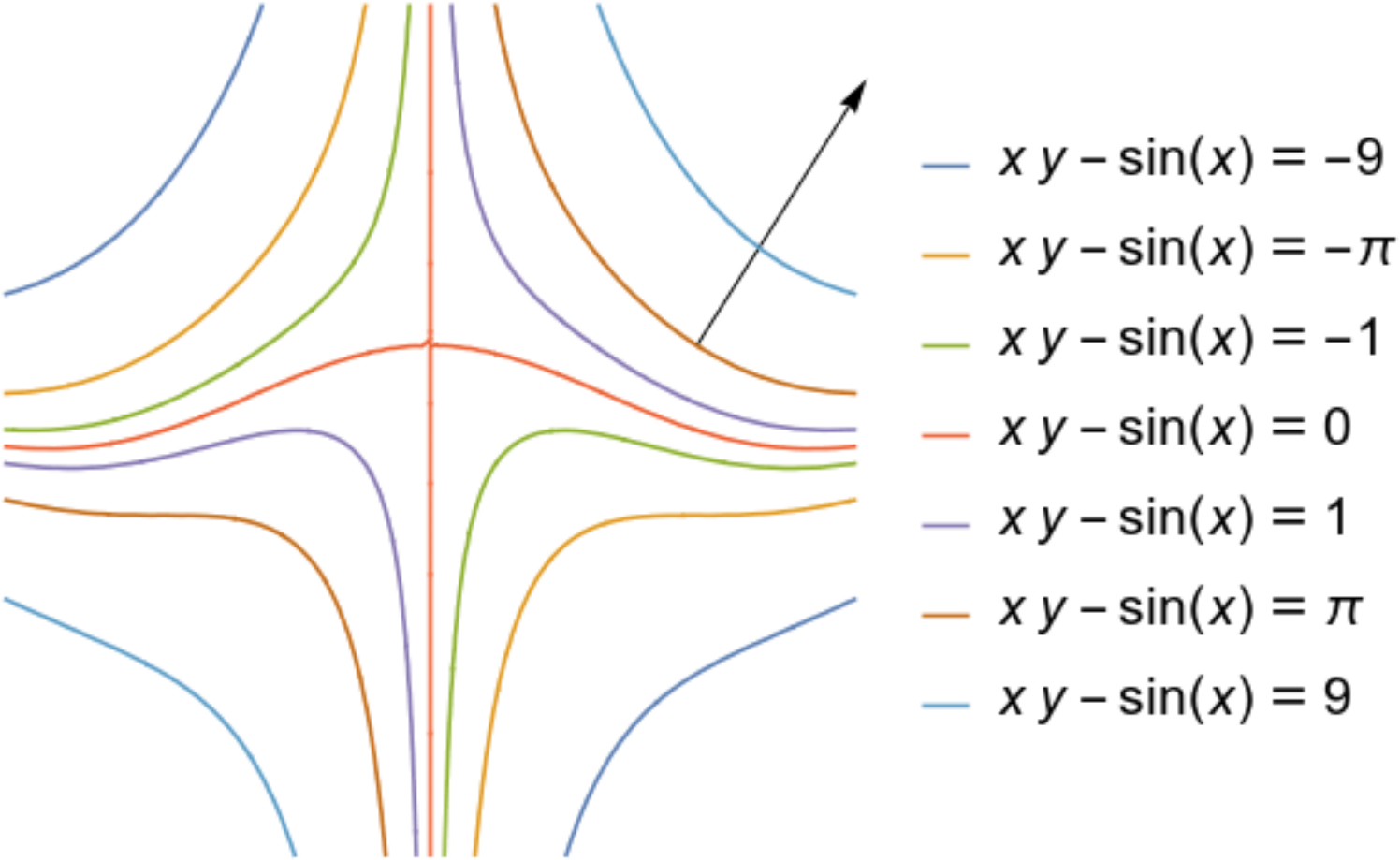

15
10
5
0
−5
−10
−15
−5
0
5
10
15
$\frac{1}{2}(-x-1)+3=-4$
$\frac{1}{2}(-x-1)+3=-3$
$\frac{1}{2}(-x-1)+3=-2$
$\frac{1}{2}(-x-1)+3=-1$
$\frac{1}{2}(-x-1)+3=0$
$\frac{1}{2}(-x-1)+3=1$
$\frac{1}{2}(-x-1)+3=2$
$\frac{1}{2}(-x-1)+3=3$
$\frac{1}{2}(-x-1)+3=4$

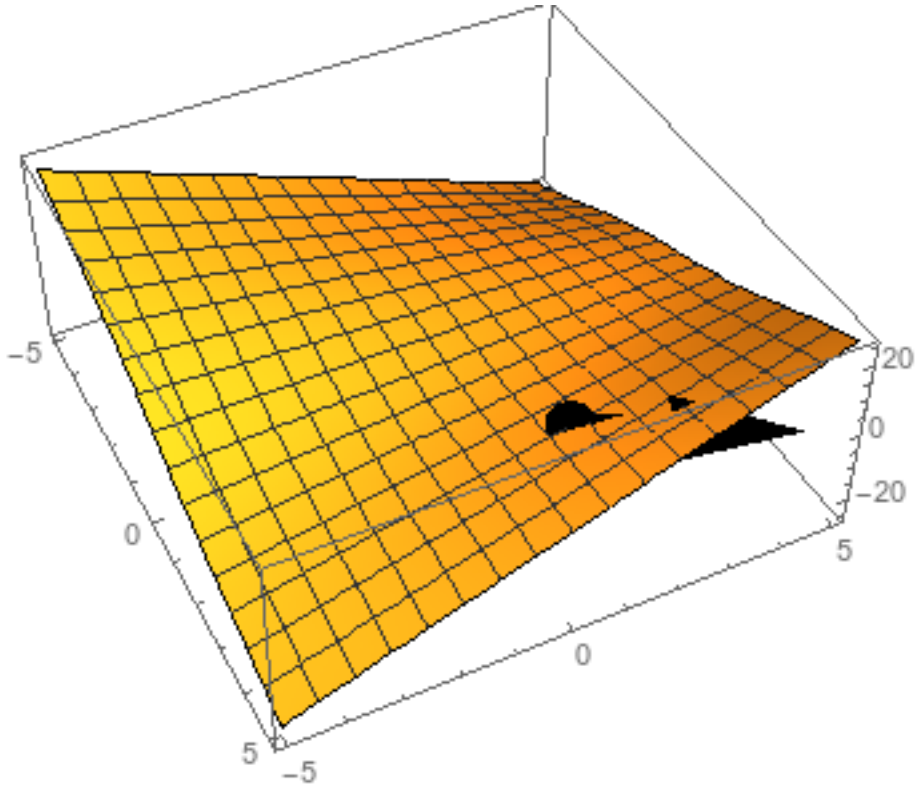

1.0
1.5
2.0
2.5
3.0
0
−10
−20
0.0
0.5
1.0
1.5
2.0

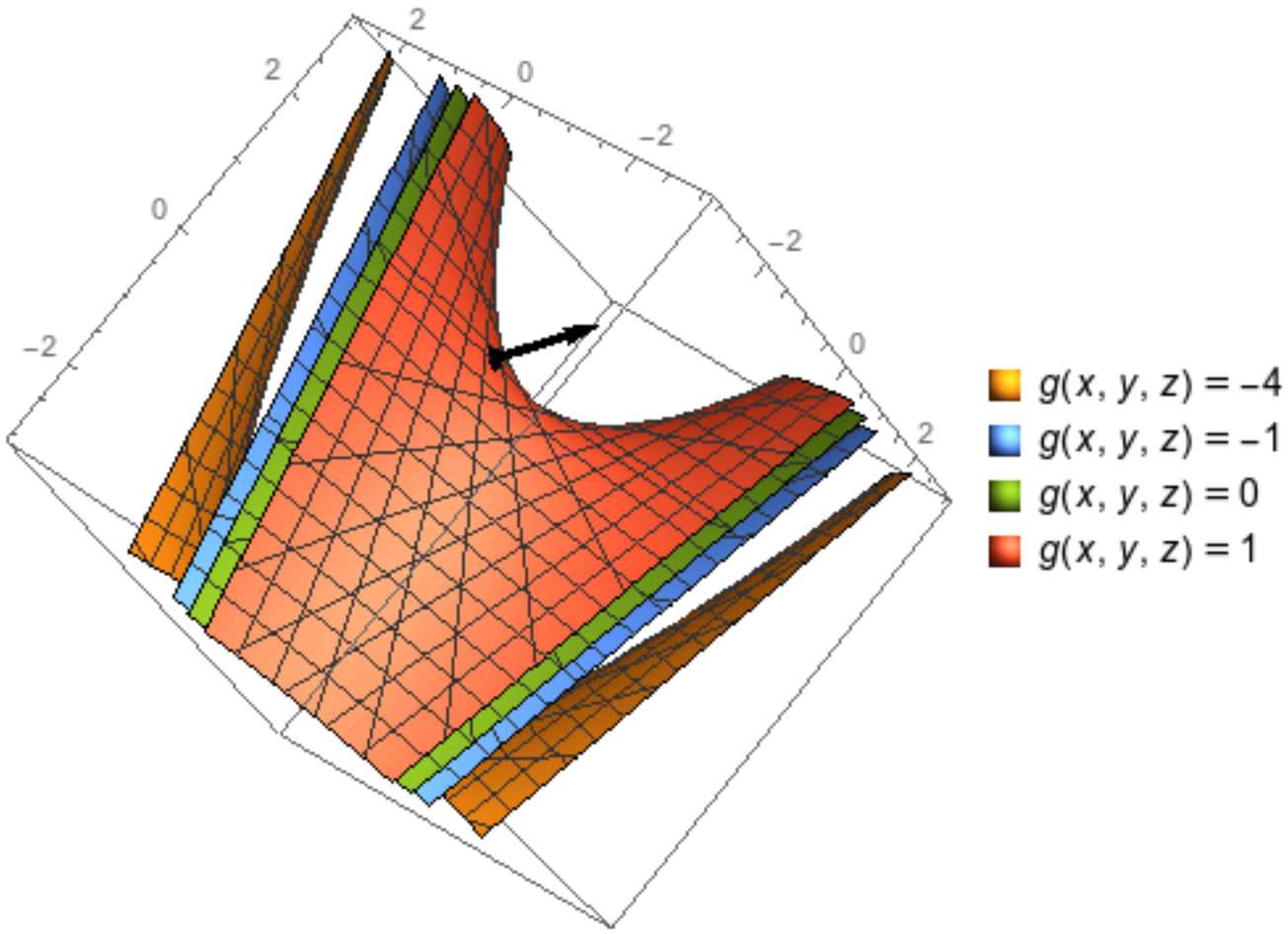

2.0
1.5
1.0
0.5
0.0
10
5
0
0.0
0.5
1.0
1.5
2.0

3.5 Directional Derivatives and the Gradient



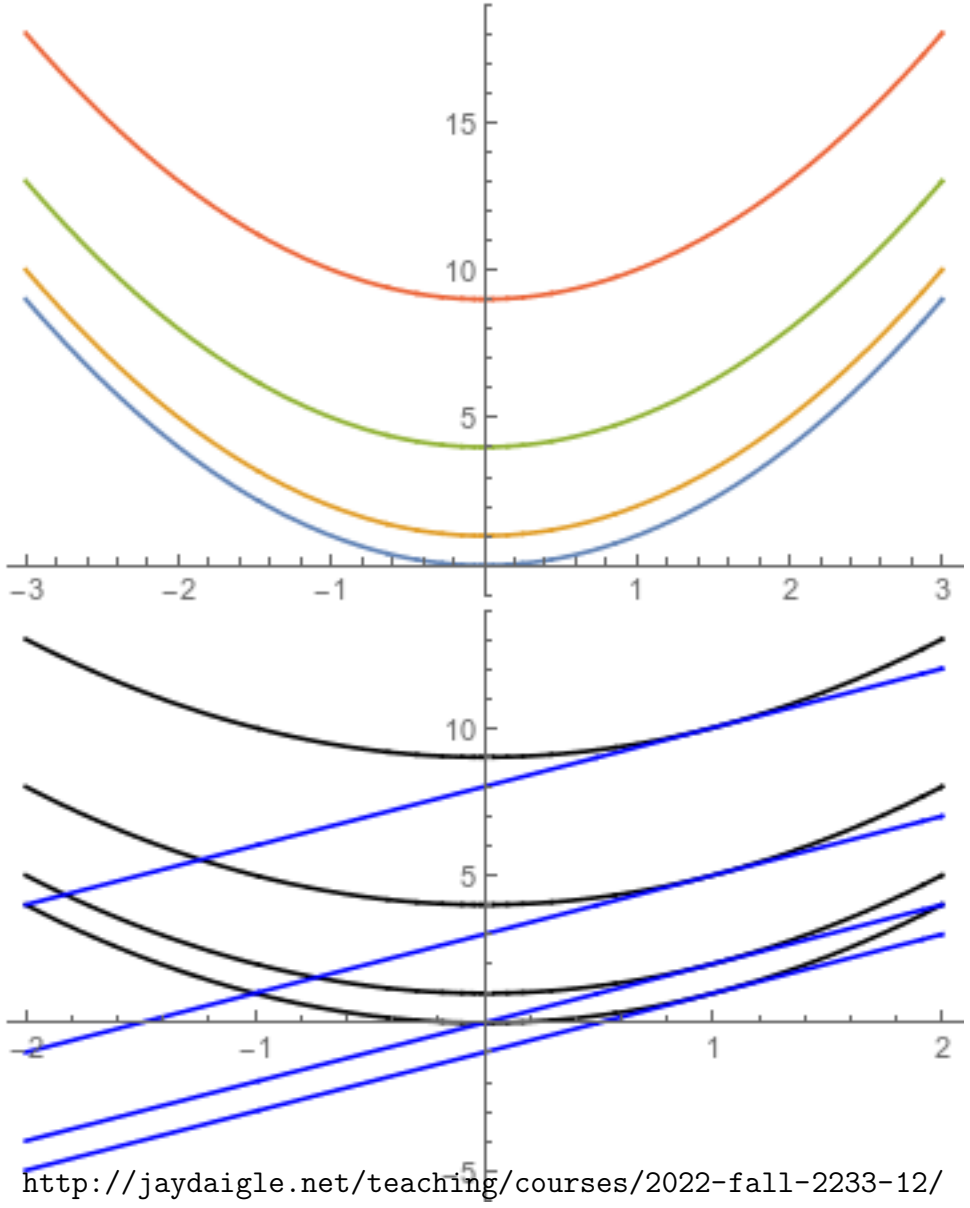

$x\,y - \sin(x) = -9$
$x\,y - \sin(x) = -\pi$
$x\,y - \sin(x) = -1$
$x\,y - \sin(x) = 0$
$x\,y - \sin(x) = 1$
$x\,y - \sin(x) = \pi$
$x\,y - \sin(x) = 9$

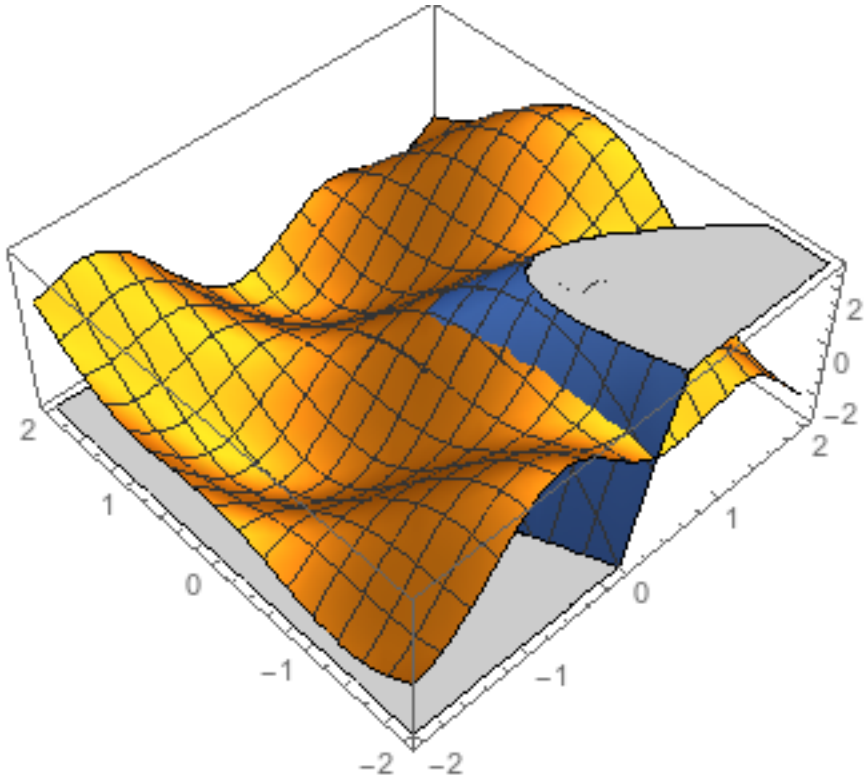

−5
0
5
−5
0
5
20
0
−20

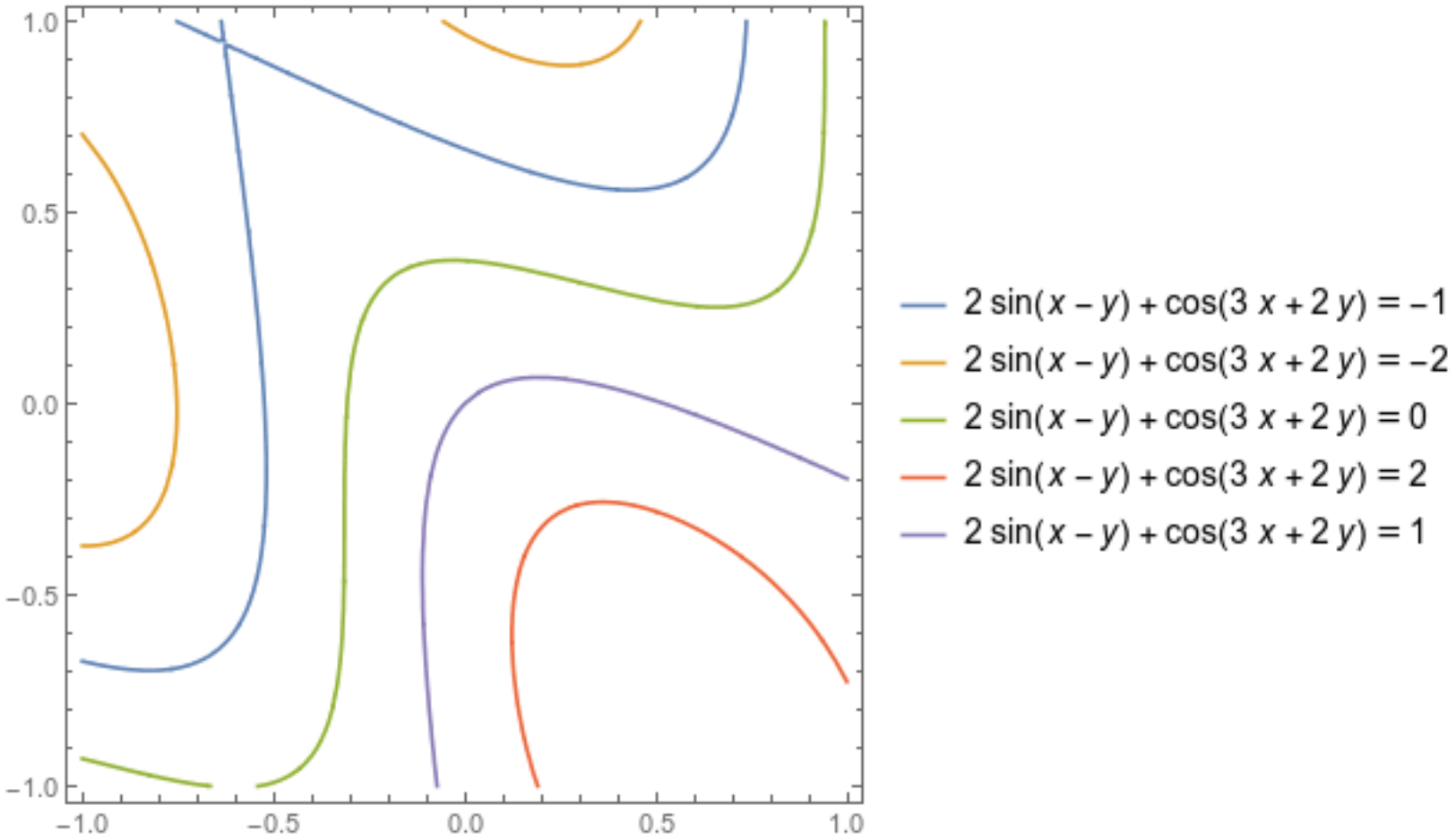

2
0
−2
2
0
−2
2
0
−2
g(x, y, z) = −4
g(x, y, z) = −1
g(x, y, z) = 0
g(x, y, z) = 1

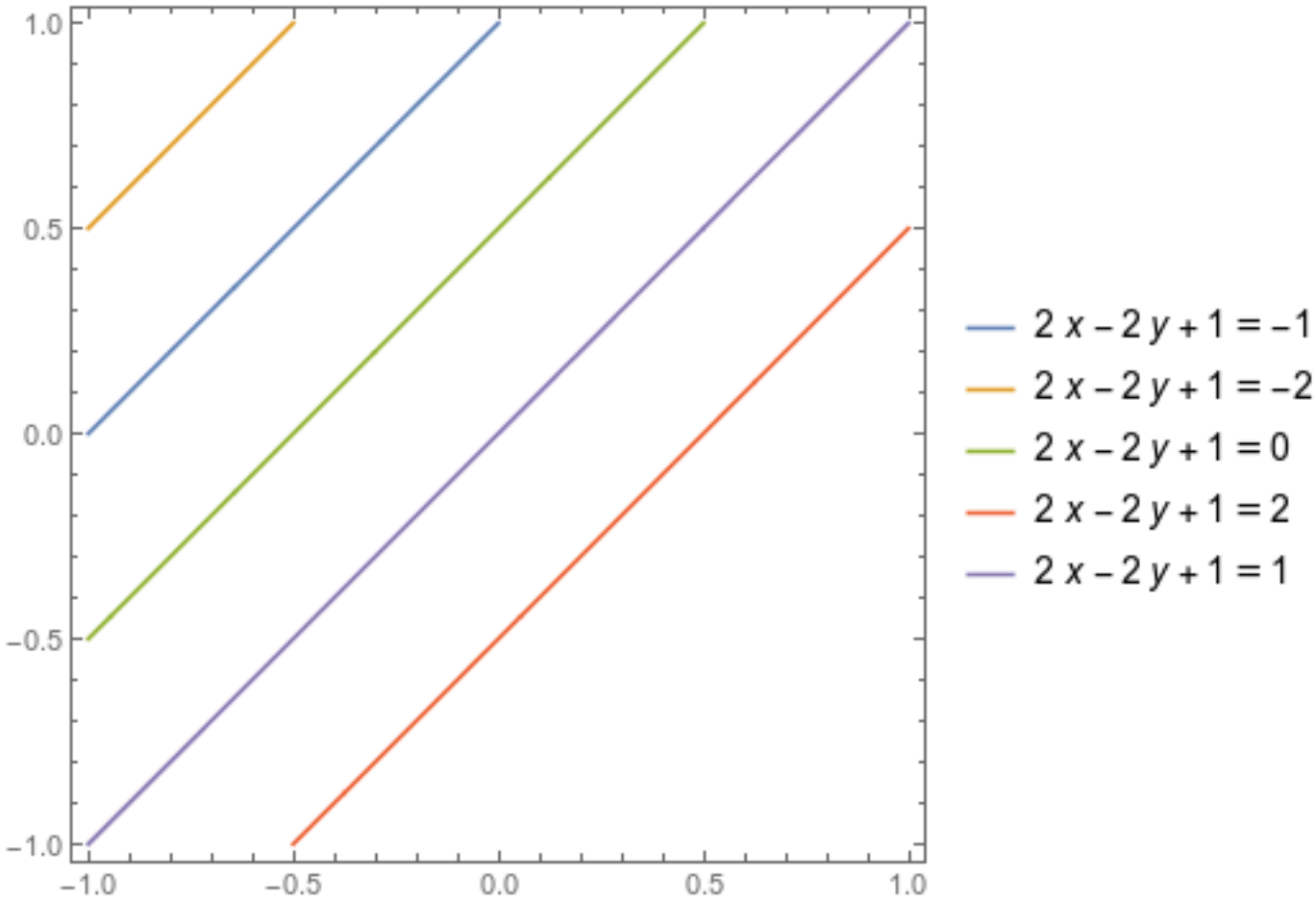
3.6 Chain Rule

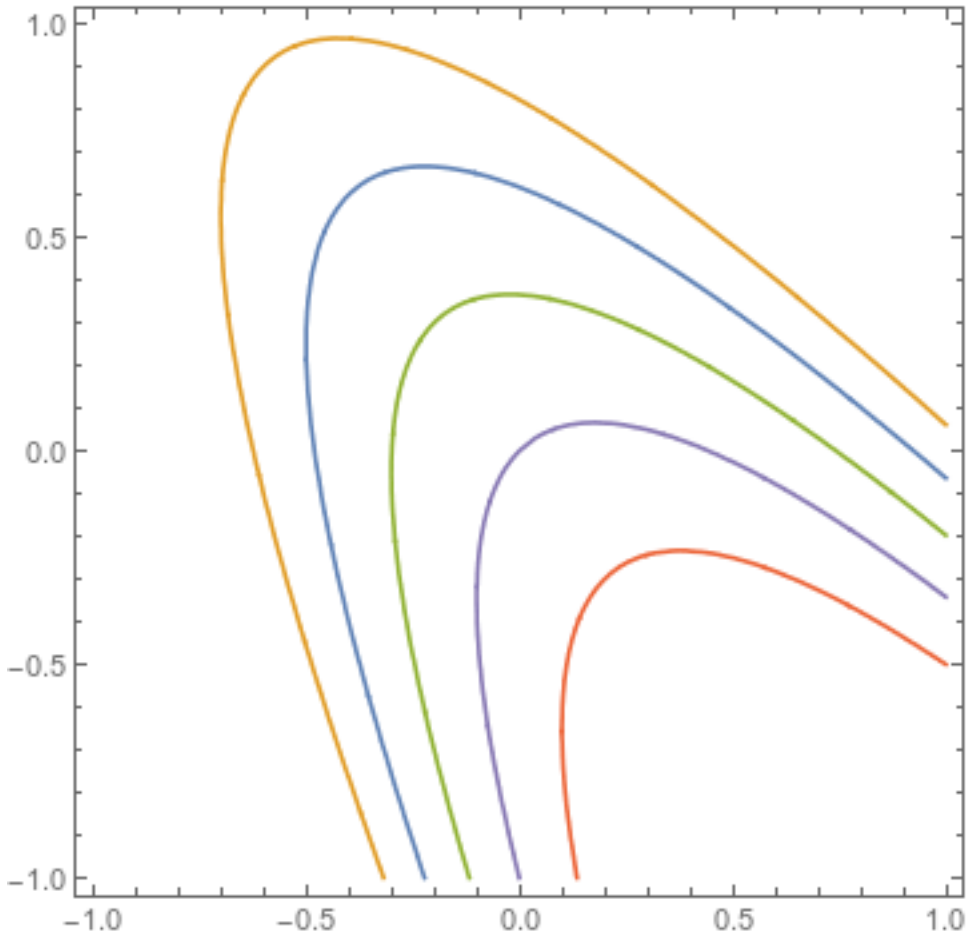
3.7 Second Partials




2
1
0
−1
−2
−2
−1
0
1
2
2
0
−2


1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
2 sin(x − y) + cos(3 x + 2 y) = −1
2 sin(x − y) + cos(3 x + 2 y) = −2
2 sin(x − y) + cos(3 x + 2 y) = 0
2 sin(x − y) + cos(3 x + 2 y) = 2
2 sin(x − y) + cos(3 x + 2 y) = 1


1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
2 x − 2 y + 1 = −1
2 x − 2 y + 1 = −2
2 x − 2 y + 1 = 0
2 x − 2 y + 1 = 2
2 x − 2 y + 1 = 1



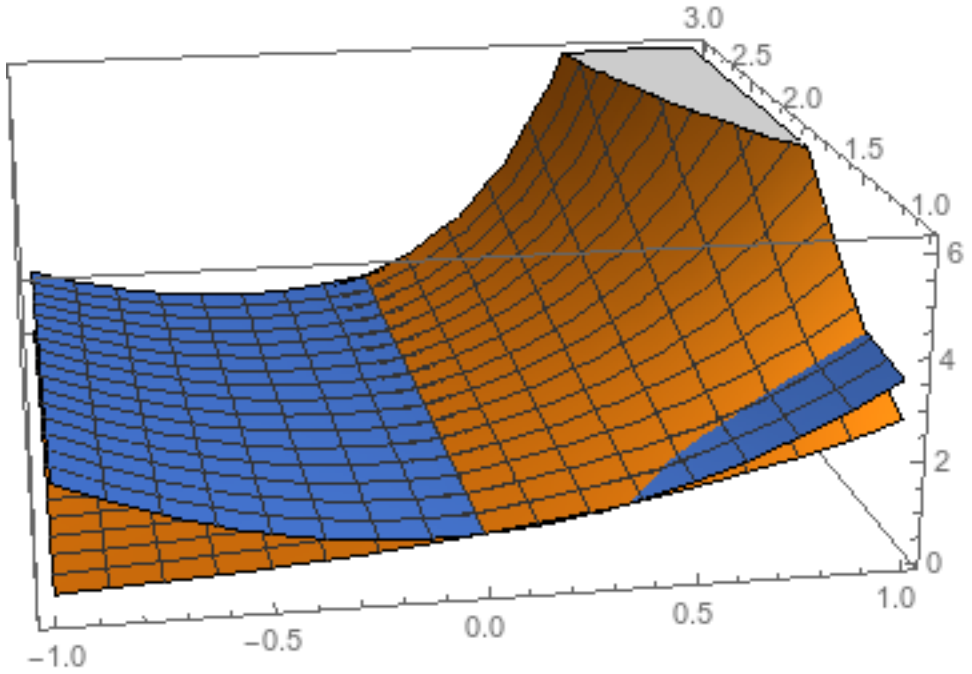
$$-\frac{9x^2}{2} - 6xy + 2x - 2y^2 - 2y + 1 = -1$$

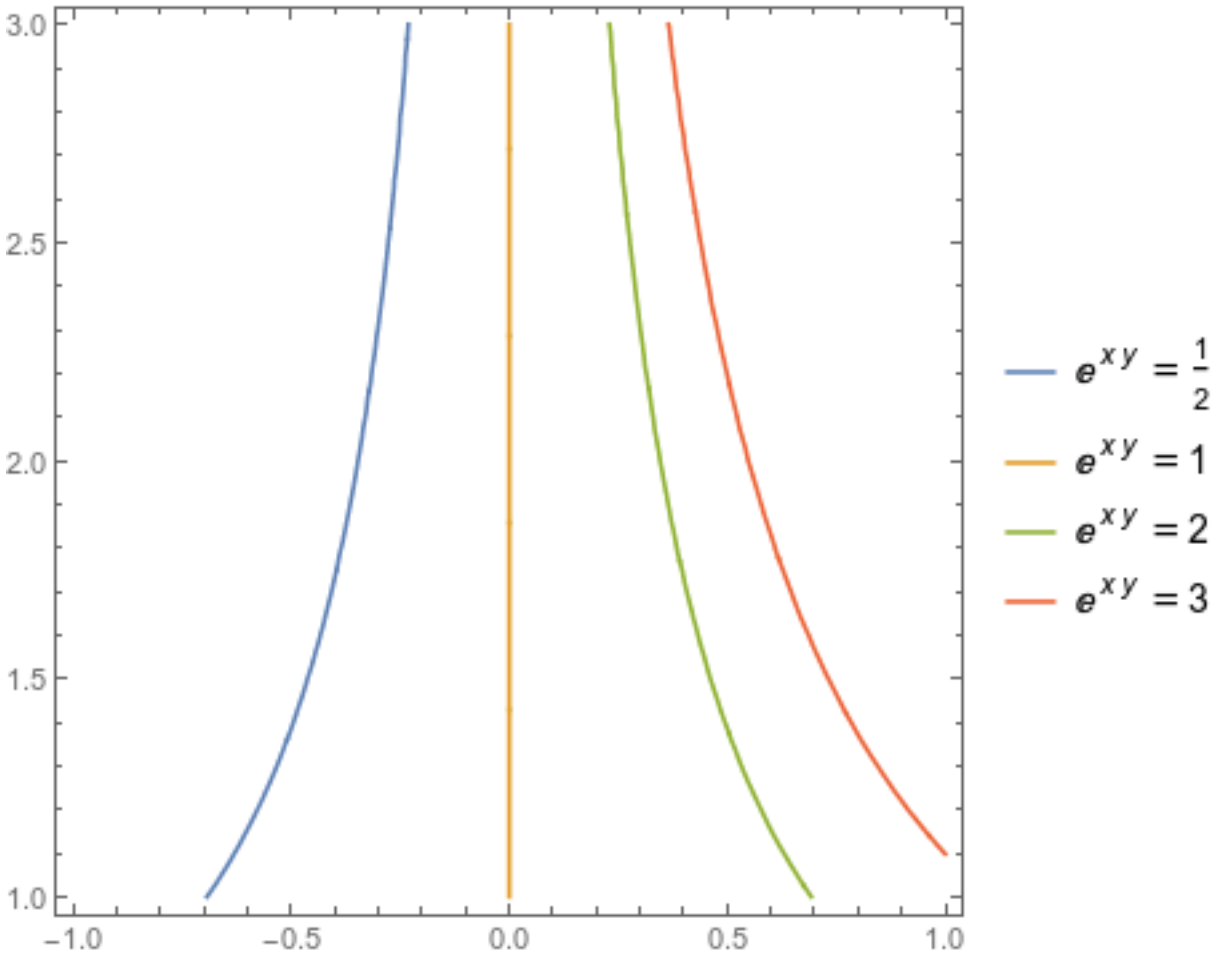
$$-\frac{9x^2}{2} - 6xy + 2x - 2y^2 - 2y + 1 = -2$$

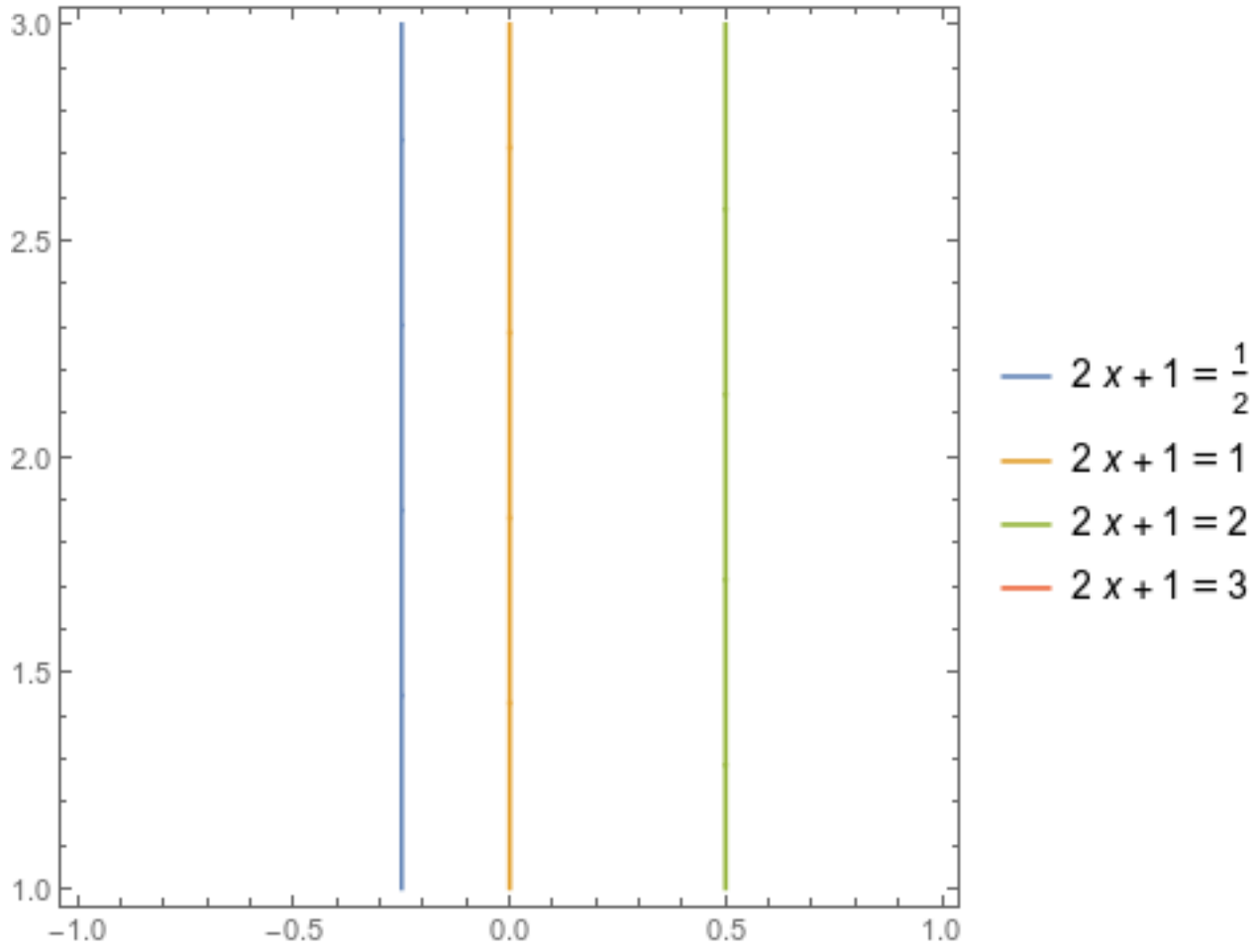
$$-\frac{9x^2}{2} - 6xy + 2x - 2y^2 - 2y + 1 = 0$$

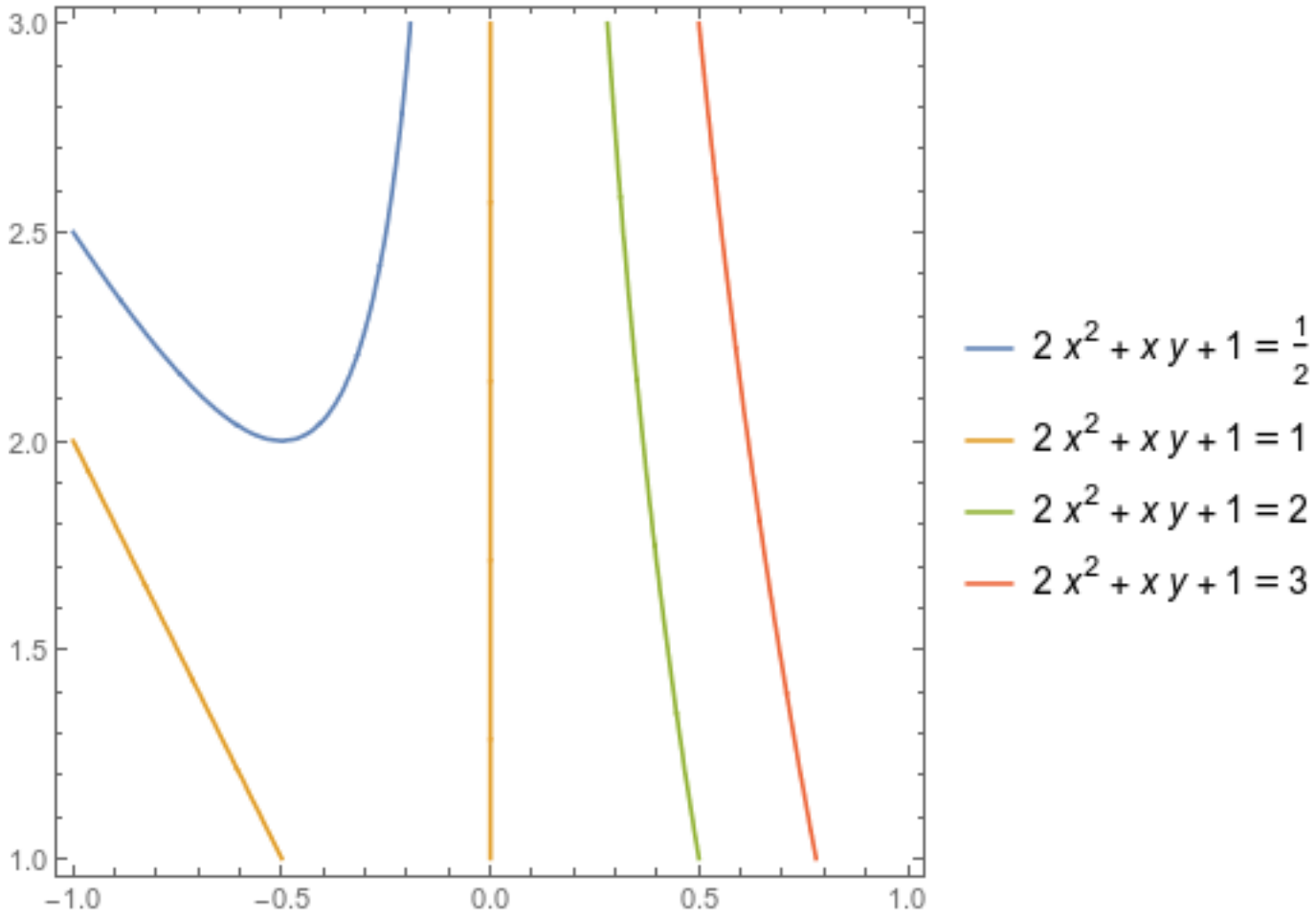
$$-\frac{9x^2}{2} - 6xy + 2x - 2y^2 - 2y + 1 = 2$$

$$-\frac{9x^2}{2} - 6xy + 2x - 2y^2 - 2y + 1 = 1$$


3.0
2.5
2.0
1.5
1.0
6
4
2
0
1.0
0.5
0.0
−0.5
−1.0


3.0
2.5
2.0
1.5
1.0
−1.0
−0.5
0.0
0.5
1.0
$e^{xy} = \frac{1}{2}$
$e^{xy} = 1$
$e^{xy} = 2$
$e^{xy} = 3$


3.0
2.5
2.0
1.5
1.0
−1.0
−0.5
0.0
0.5
1.0
2 x + 1 = 1/2
2 x + 1 = 1
2 x + 1 = 2
2 x + 1 = 3


3.0
2.5
2.0
1.5
1.0
−1.0
−0.5
0.0
0.5
1.0
$2x^2 + xy + 1 = \frac{1}{2}$
$2x^2 + xy + 1 = 1$
$2x^2 + xy + 1 = 2$
$2x^2 + xy + 1 = 3$