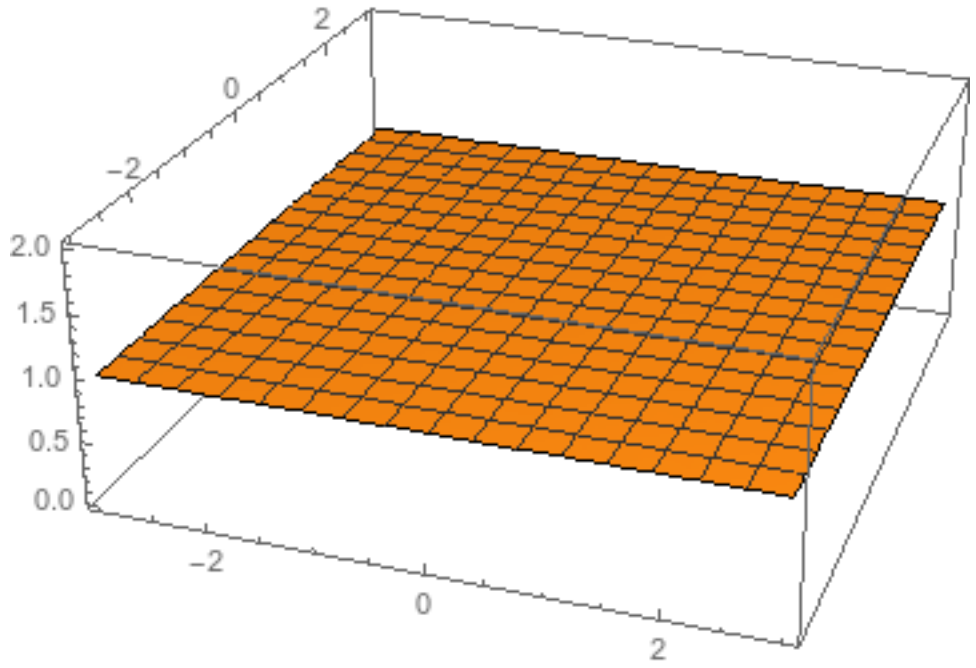
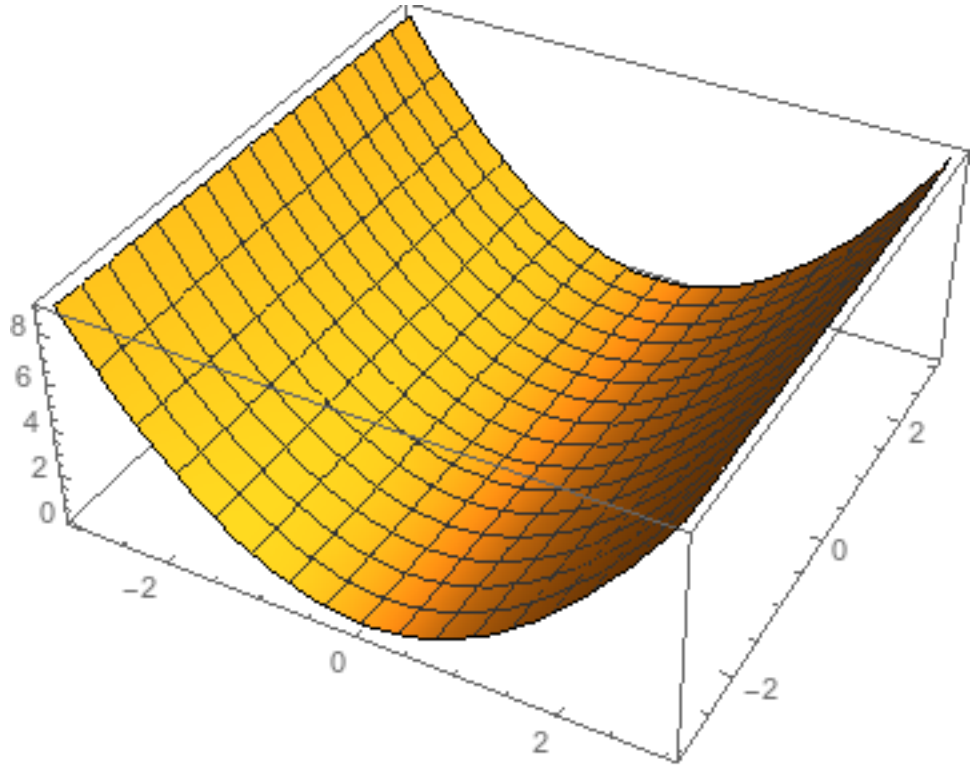
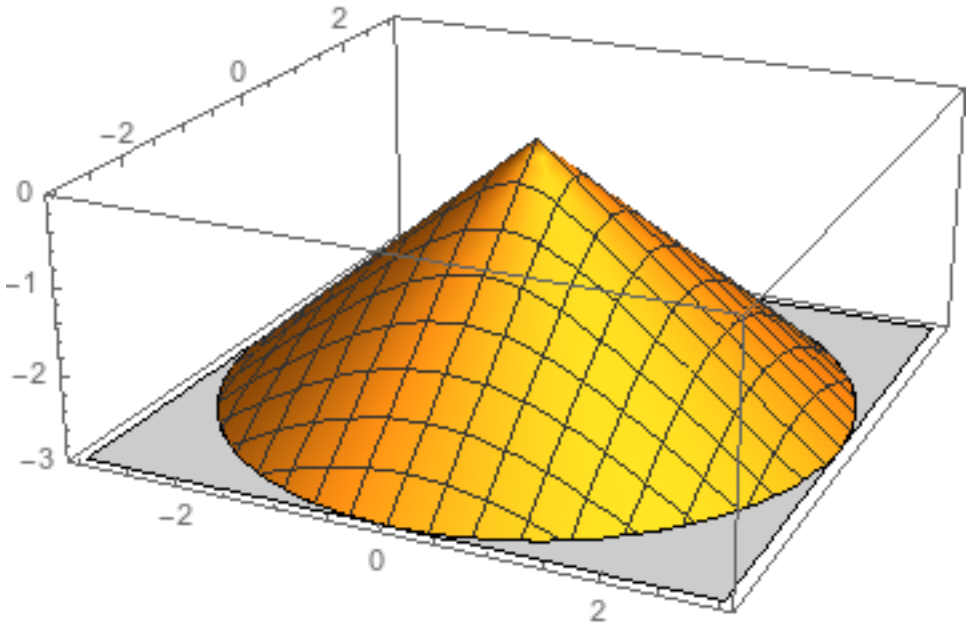
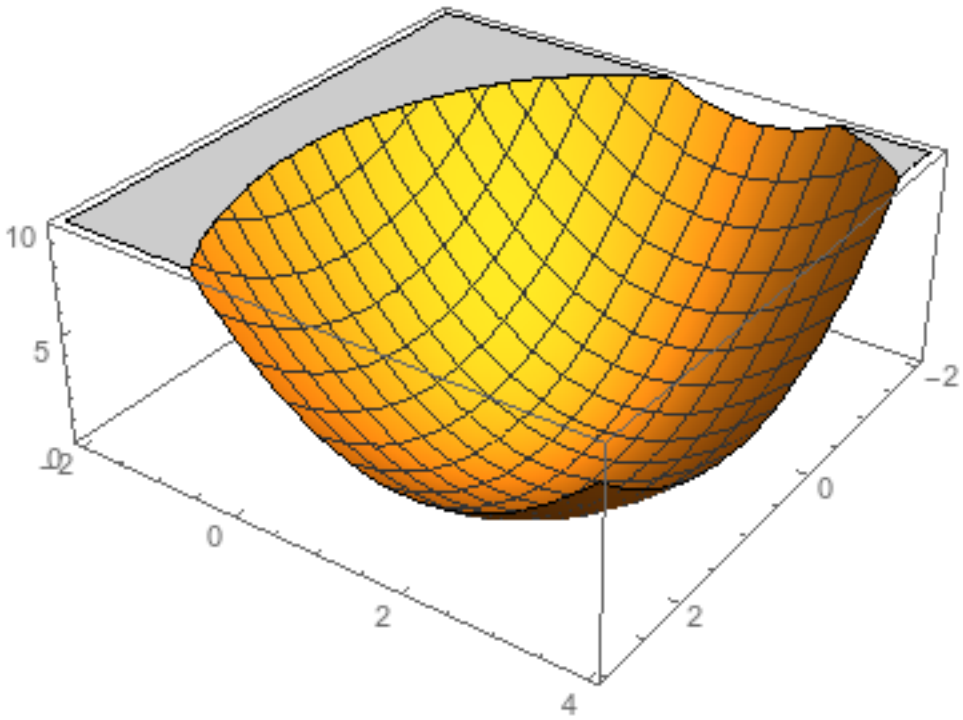
4 Optimization

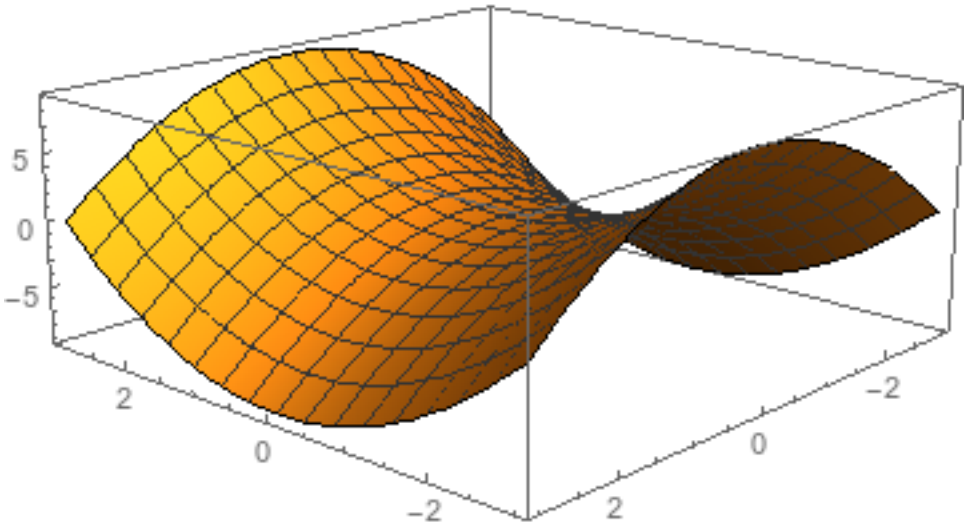
4.1 Critical Points and Local Extrema

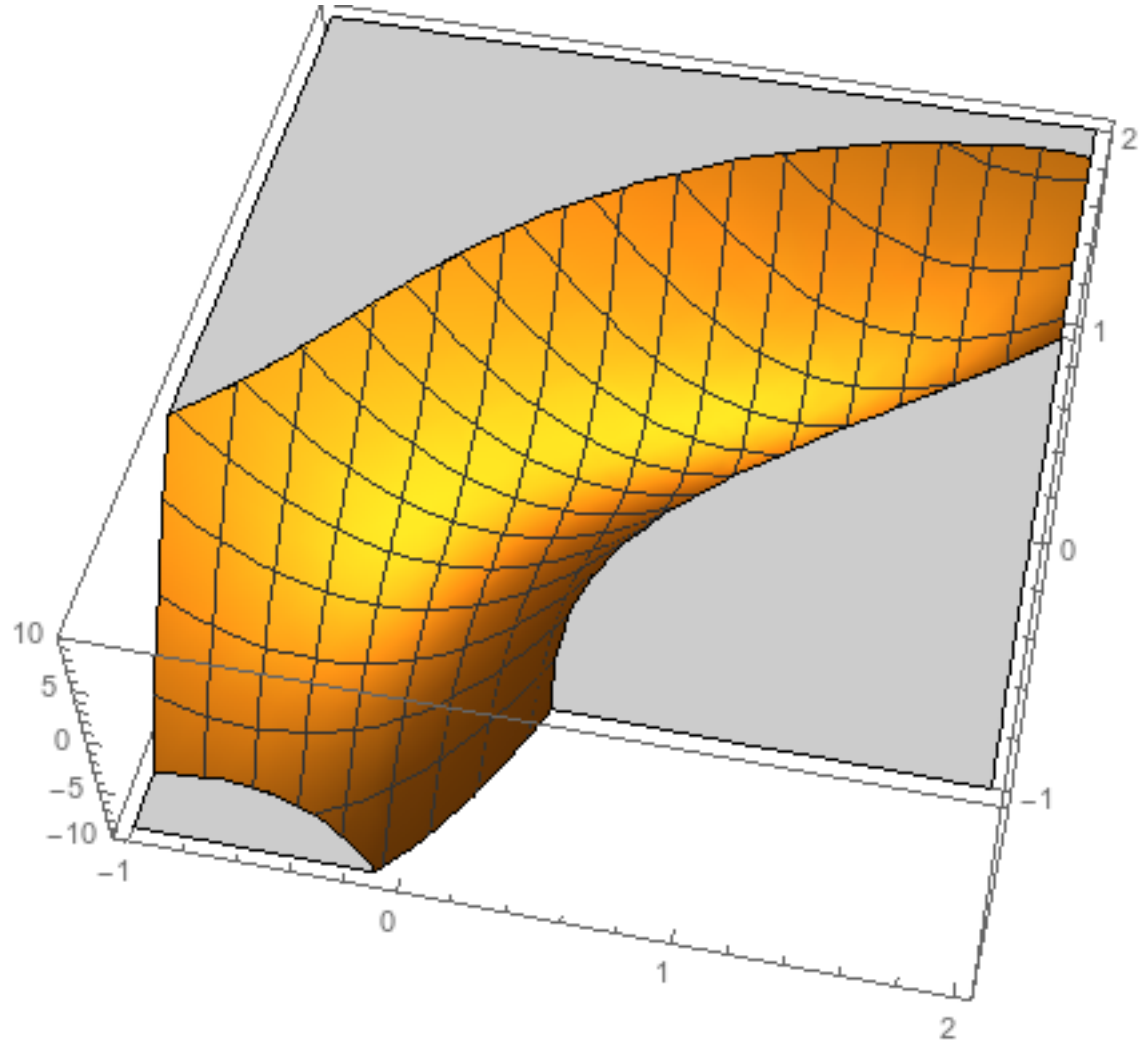


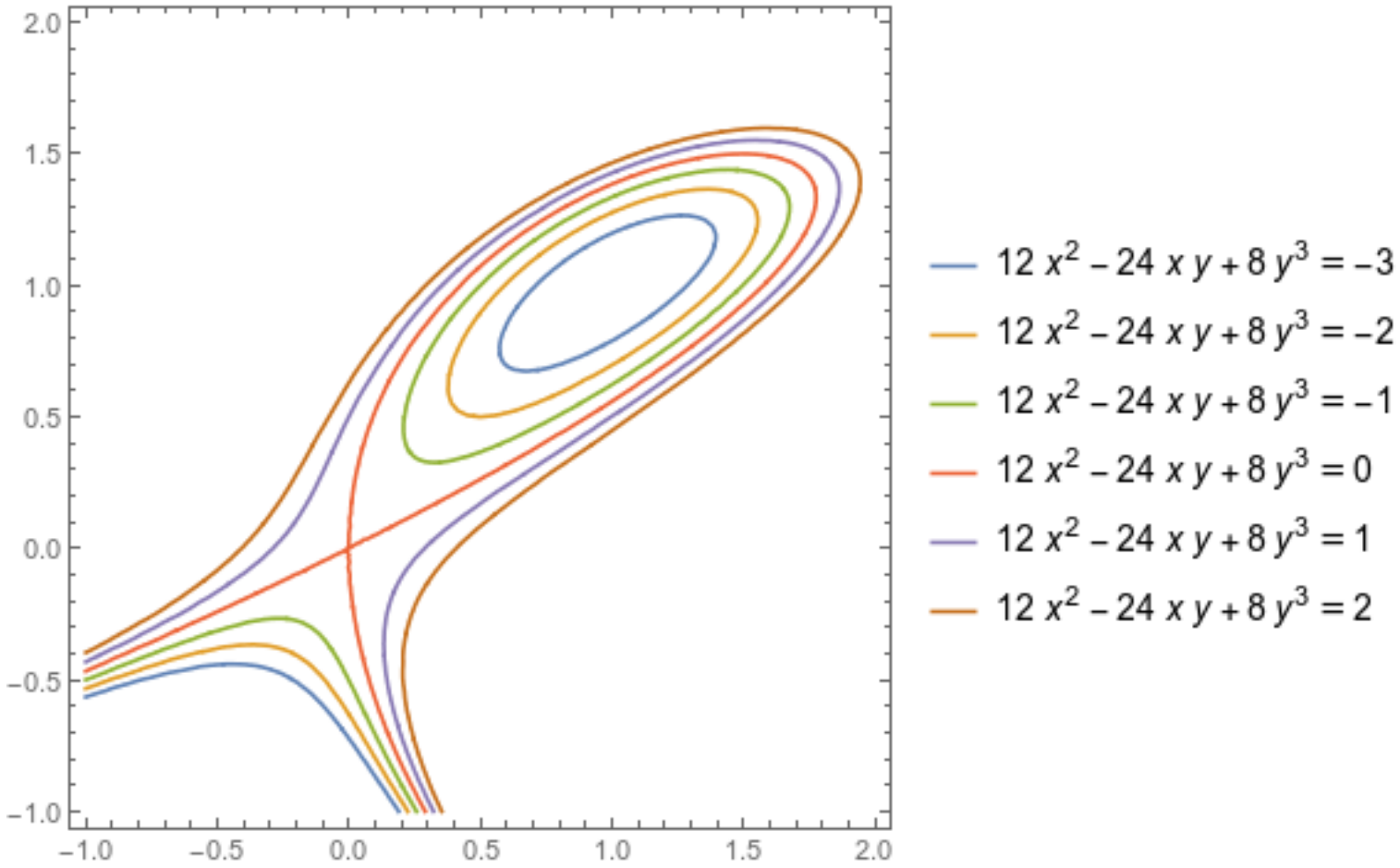

8
6
4
2
0
−2
0
2
2
0
−2

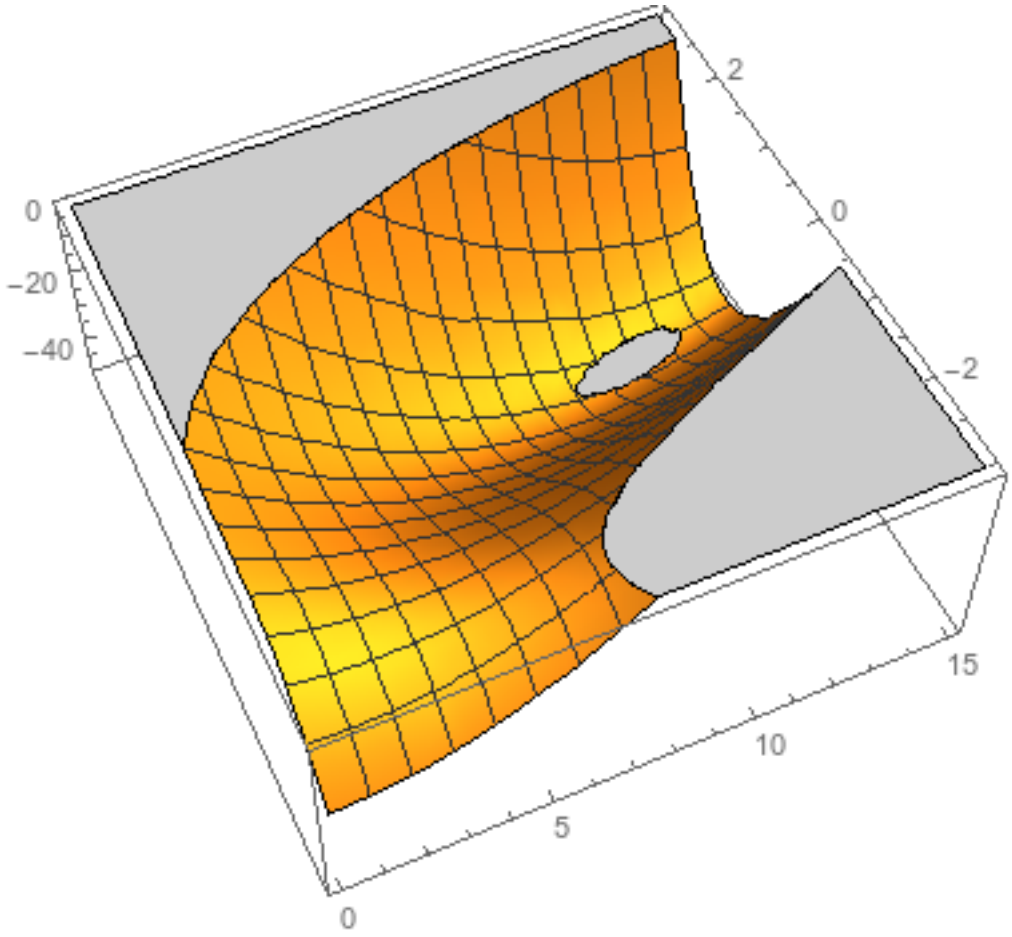

2
0
−2
0
−1
−2
−3
−2
0
2

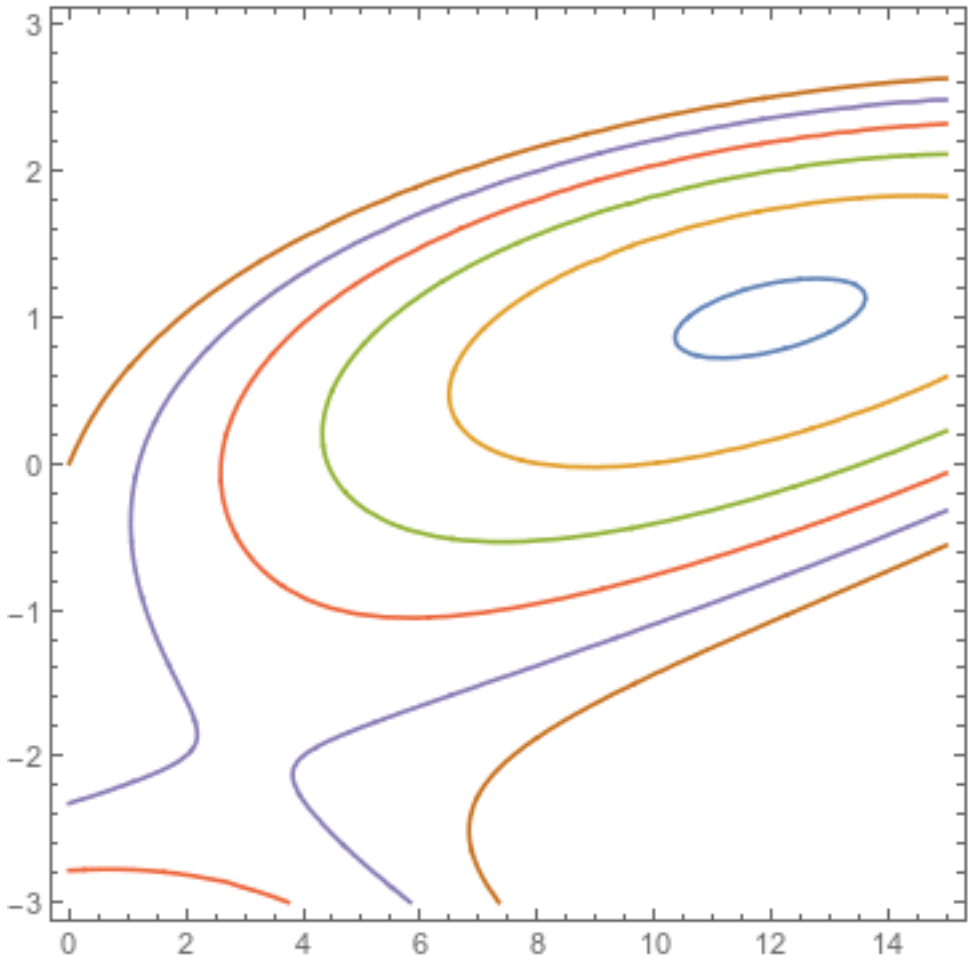

10
5
−2
0
2
0
2
4
0
2
−2


5
0
−5
2
0
−2
2
0
−2


10
5
0
−5
−10
−1
0
1
2
2
1
0
−1


2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
$12\,x^2 - 24\,x\,y + 8\,y^3 = -3$
$12\,x^2 - 24\,x\,y + 8\,y^3 = -2$
$12\,x^2 - 24\,x\,y + 8\,y^3 = -1$
$12\,x^2 - 24\,x\,y + 8\,y^3 = 0$
$12\,x^2 - 24\,x\,y + 8\,y^3 = 1$
$12\,x^2 - 24\,x\,y + 8\,y^3 = 2$


0
−20
−40
2
0
−2
0
5
10
15



$-3xy + \frac{x^2}{2} - 9x + 3y^3 + 9y^2 + 9y = -50$

$-3xy + \frac{x^2}{2} - 9x + 3y^3 + 9y^2 + 9y = -40$

$-3xy + \frac{x^2}{2} - 9x + 3y^3 + 9y^2 + 9y = -30$

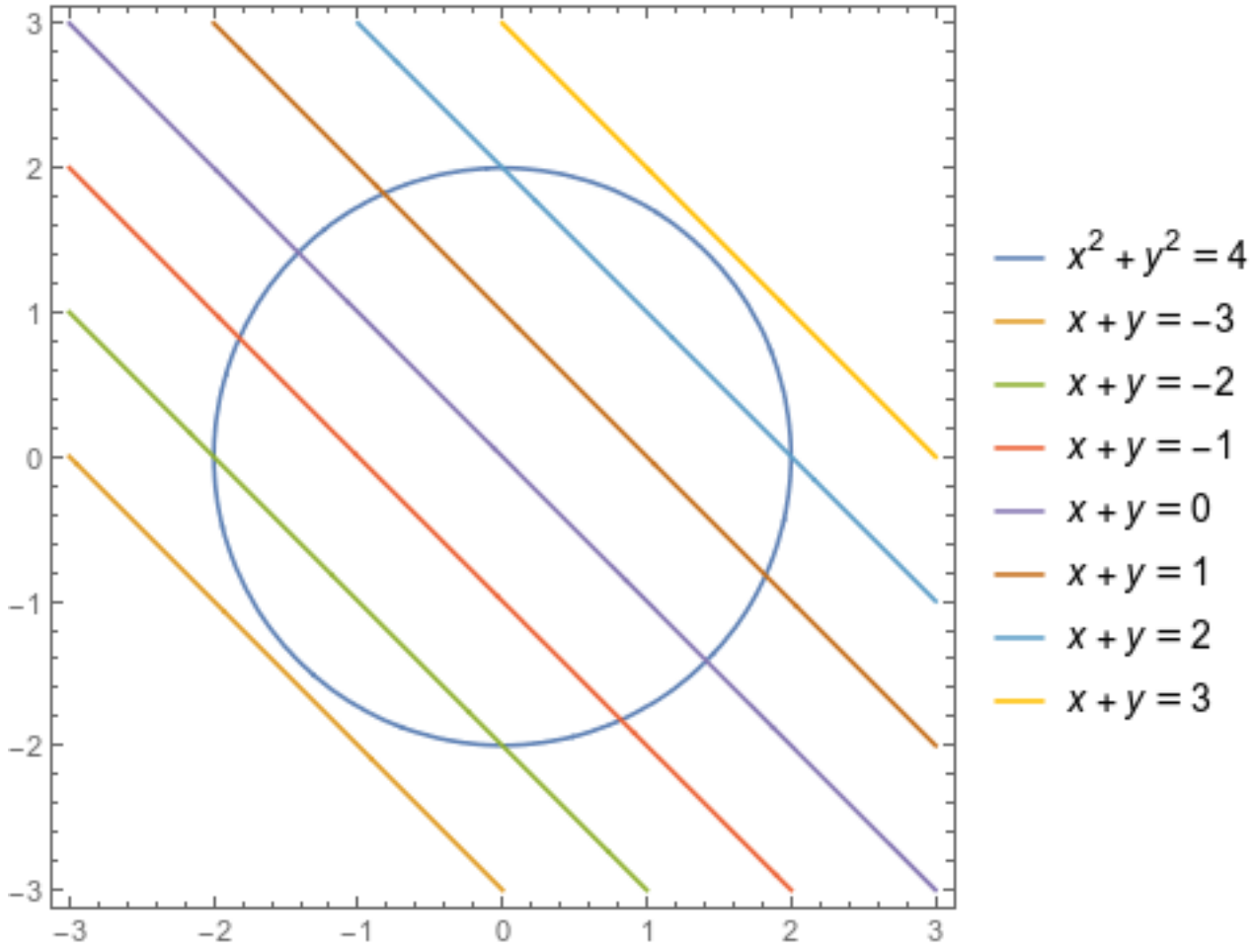
$-3xy + \frac{x^2}{2} - 9x + 3y^3 + 9y^2 + 9y = -20$

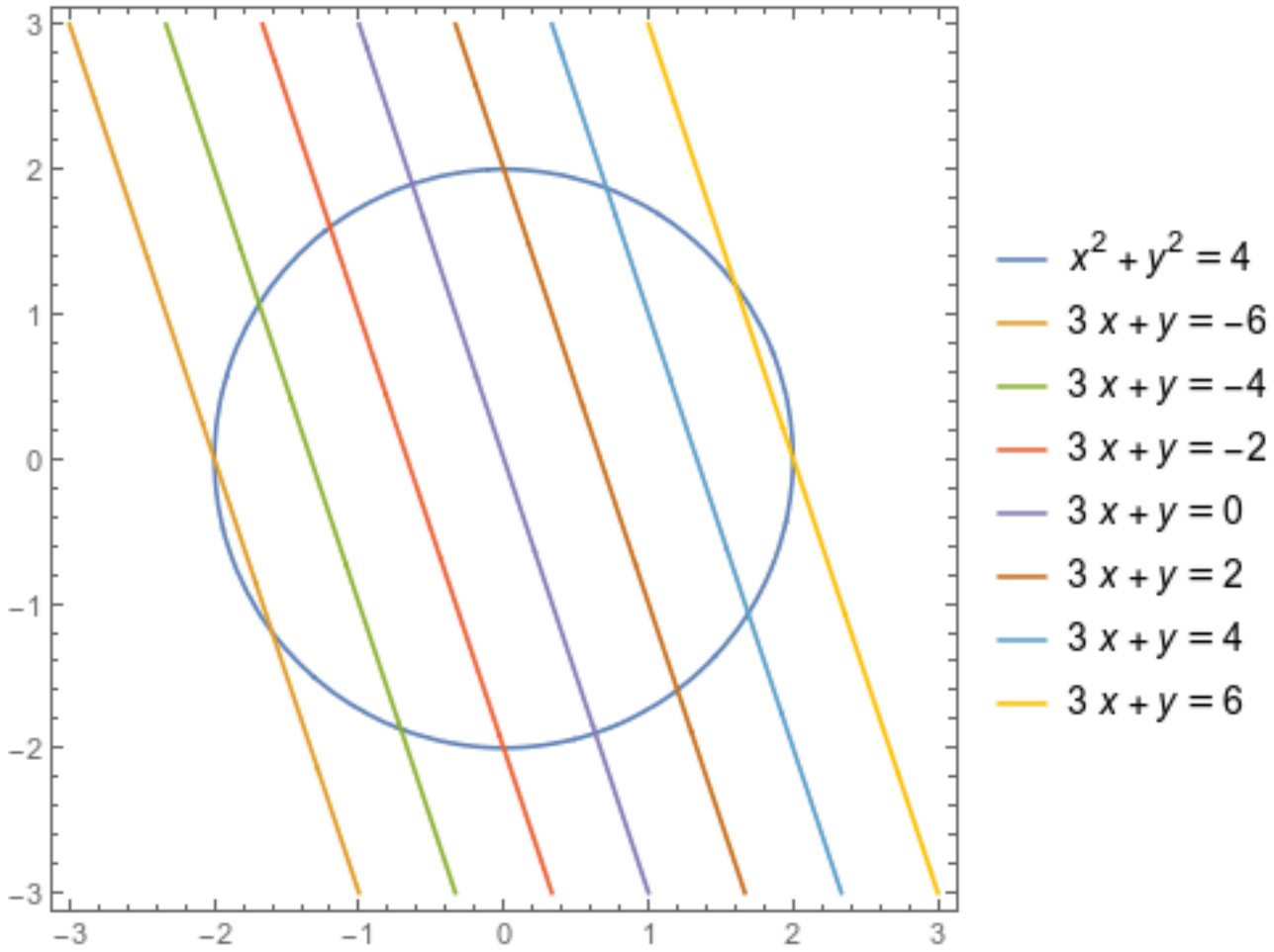
$-3xy + \frac{x^2}{2} - 9x + 3y^3 + 9y^2 + 9y = -10$

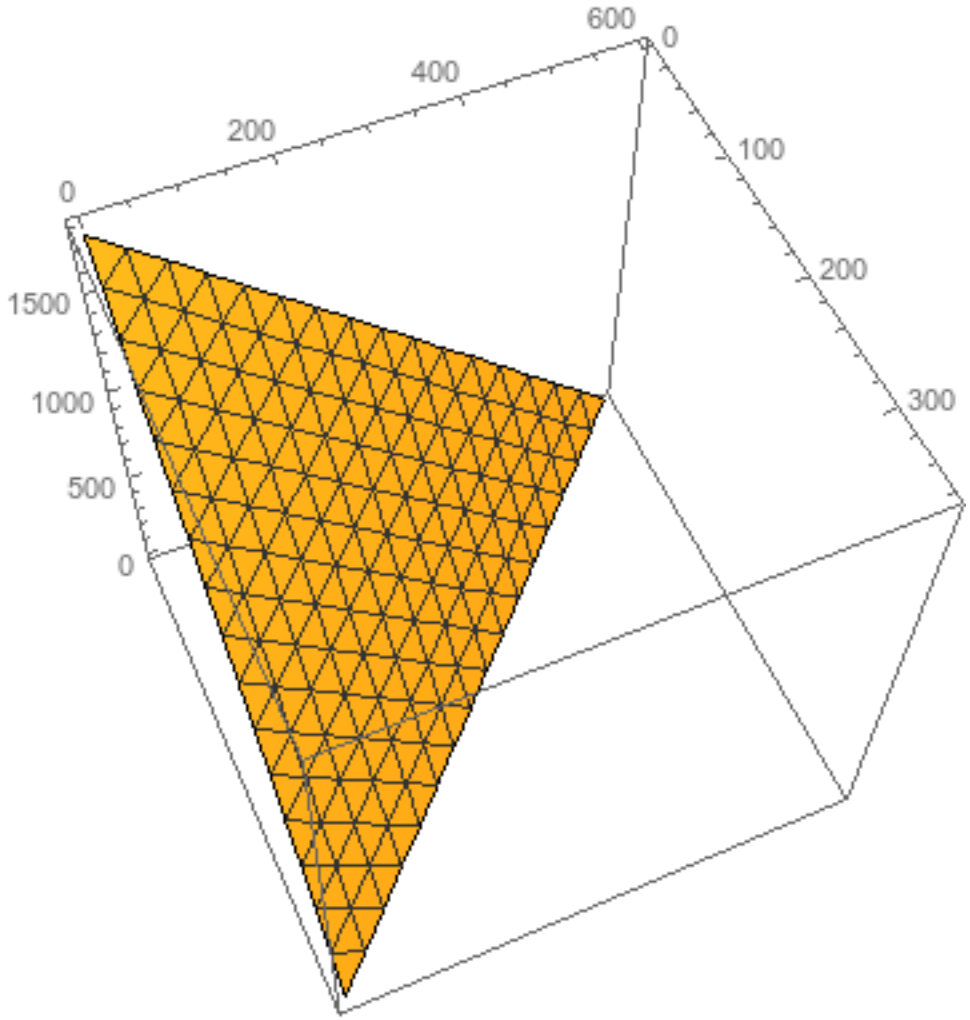
$-3xy + \frac{x^2}{2} - 9x + 3y^3 + 9y^2 + 9y = 0$

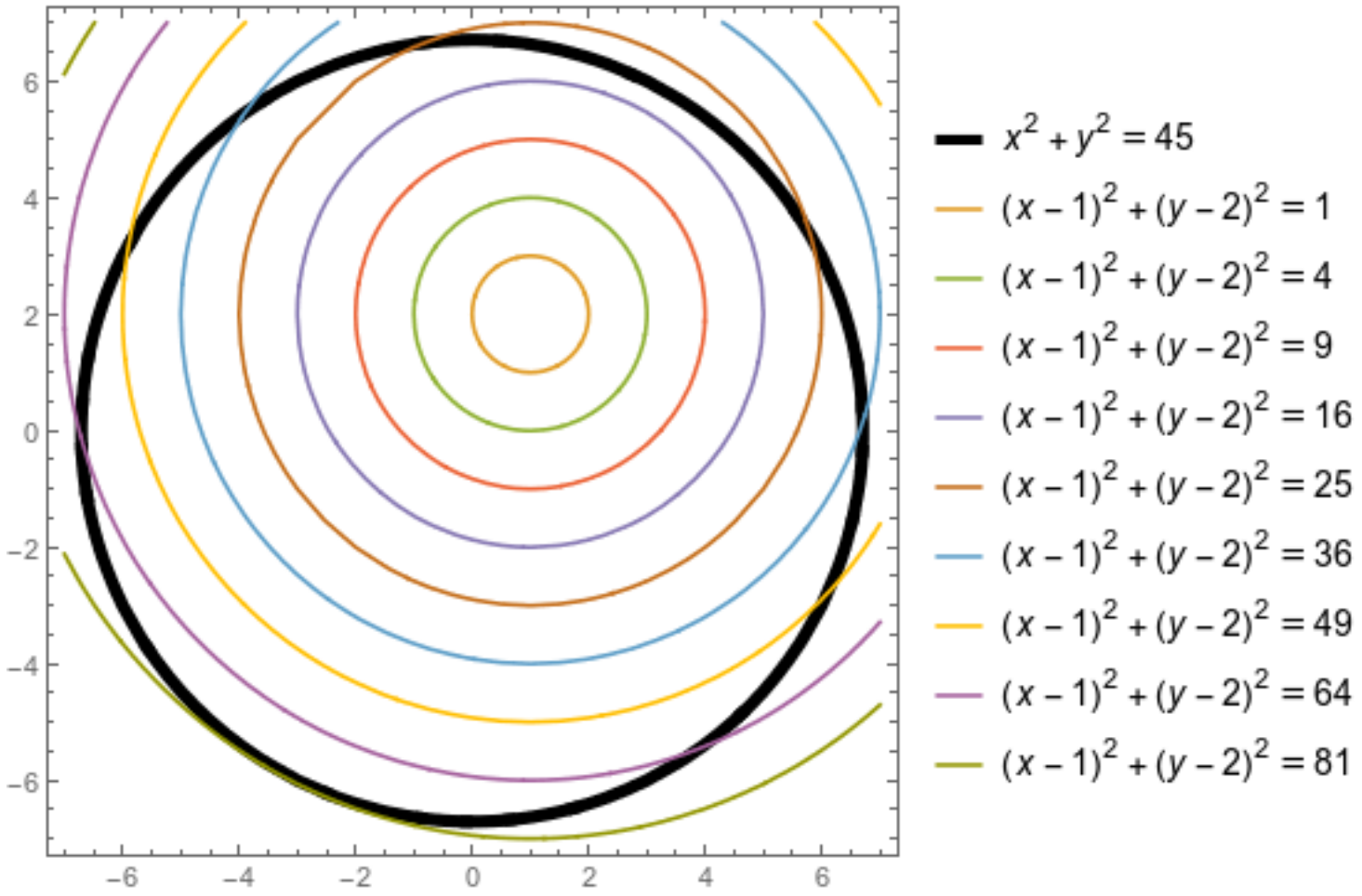
4.2 Global Extrema and the Extreme Value Theorem

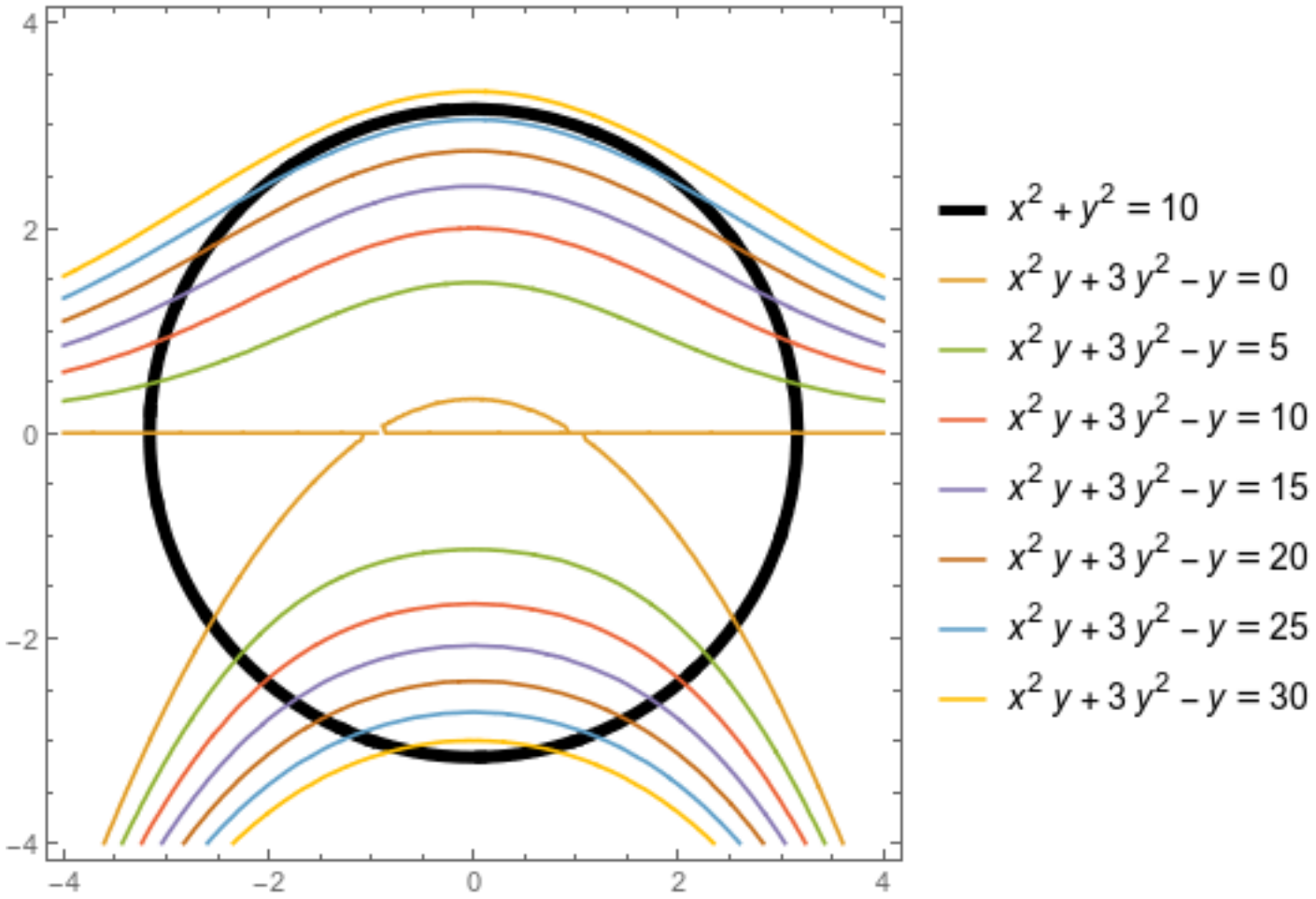
4.3 Constrained Optimization and Lagrange Multipliers




3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
$x^2 + y^2 = 4$
$3x + y = -6$
$3x + y = -4$
$3x + y = -2$
$3x + y = 0$
$3x + y = 2$
$3x + y = 4$
$3x + y = 6$


600
400
200
0
0
100
200
300
1500
1000
500
0


6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
$x^2 + y^2 = 45$
$(x-1)^2 + (y-2)^2 = 1$
$(x-1)^2 + (y-2)^2 = 4$
$(x-1)^2 + (y-2)^2 = 9$
$(x-1)^2 + (y-2)^2 = 16$
$(x-1)^2 + (y-2)^2 = 25$
$(x-1)^2 + (y-2)^2 = 36$
$(x-1)^2 + (y-2)^2 = 49$
$(x-1)^2 + (y-2)^2 = 64$
$(x-1)^2 + (y-2)^2 = 81$


4
2
0
−2
−4
−4
−2
0
2
4
$x^2 + y^2 = 10$
$x^2\,y + 3\,y^2 - y = 0$
$x^2\,y + 3\,y^2 - y = 5$
$x^2\,y + 3\,y^2 - y = 10$
$x^2\,y + 3\,y^2 - y = 15$
$x^2\,y + 3\,y^2 - y = 20$
$x^2\,y + 3\,y^2 - y = 25$
$x^2\,y + 3\,y^2 - y = 30$