# Math 2233 Practice Final Solutions

Instructor: Jay Daigle

- You will have 90 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, two-sided, handwritten cheat sheet you have made for yourself ahead of time. You may not use a calculator.
- The exam has 7 problems, one on each mastery topic. The exam has 4 pages total.
- Each part of each major topic is worth 10 points. The question on topic S5 is worth 10 points.

  This practice test has too many questions so you can get in a broad spectrum of practice. I expect one question per topic for M1 through M4, and two questions on M5 and M6, on the real final.
- The real final will have optional questions on S1 through S4. Answering one correctly can earn you up to two bonus points on the test. More importantly, answering one correctly can raise your overall mastery score.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

  When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

## Name:

## Recitation Section:

**Problem 1** (M1)**.** The final will have *one* problem like this.

(a) Find the area of the parallelogram with vertices $(1,3,2),(1,5,3),(2,4,5),(2,6,6)$.

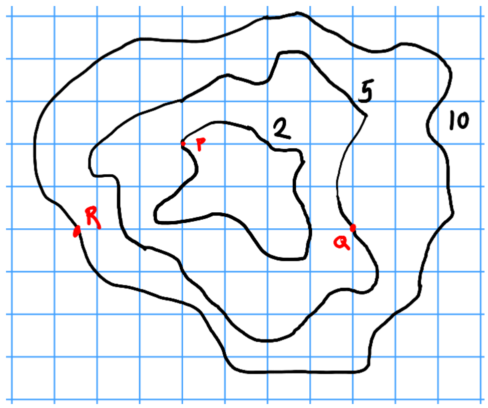
(b) Find the orthogonal decomposition of $2\vec{i}+5\vec{j}-4\vec{k}$ with respect to $5\vec{i}-\vec{j}+2\vec{k}$.

**Problem 2** (M2)**.** The final will have *one* problem like this.

(a) Find a linear approximation of $f(x,y)=\sin(x)\sqrt{1-y^2}$ near the point $(0,0)$. Use it to estimate $f(.1,.1)$.

(b) Find an equation for the plane tangent to $g(x,y)=4xy^2+3xy$ at the point $(3,2)$.

(c) Consider the contour plot below. Estimate $\frac{\partial f}{\partial x}(Q)$ and $\frac{\partial f}{\partial y}(P)$. Sketch the gradient vector at $R$.




**Problem 3** (M3)**.** The final will have *one* problem like this.

(a) Find and classify all the critical points of $g(x,y)=x^2-3xy+5x-2y+6y^2+8$.

(b) Find the minimum value of $f(x,y)=4xy$ on the unit circle.

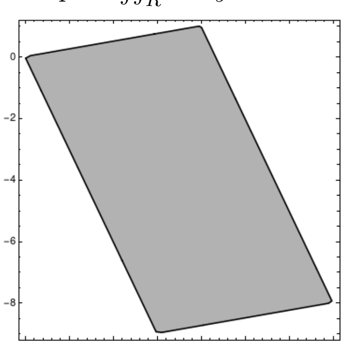
**Problem 4** (M4)**.** The final will have *one* problem like this.

Let $g(x,y,z)=z^2(x^2+y^2)$ and let $W$ be a cone with its point at the origin and base given by the circle $z=2, x^2+y^2=2$.

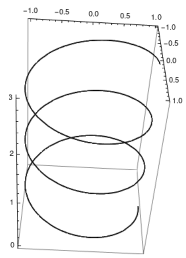
(a) Set up integrals to compute $\int_W g\,dV$ in cartesian, cylindrical, and spherical coordinates.

(b) Choose one of the integrals from part (a) and evaluate it.

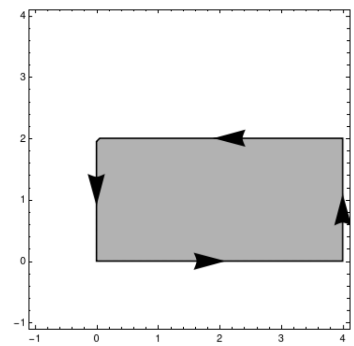
(c) Compute $\iint_R x+y\,dA$ over the parallelogram with vertices $(0,0),(4,1),(7,-8),(3,-9)$.



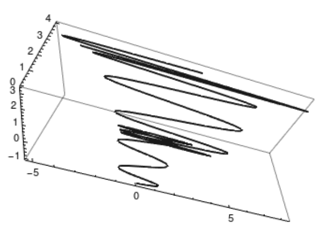
**Problem 5** (M5). (a) Set up an integral to compute the work done by the force field $\vec{F}(x^2y, yz^3, x+y+z)$ on a particle that moves from $(1,0,0)$ to $(1,0,3)$ by spiraling clockwise around the $z$-axis three times with radius 1.



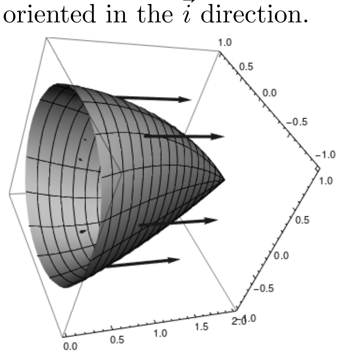
(b) Find the circulation of $\vec{F}(x,y) = -3y\vec{i} + 2x\vec{j}$ counterclockwise around the rectangle $0 \le x \le 4, 0 \le y \le 2$.



(c) Find the integral of the vector field $\vec{F}(x,y,z) = yz\vec{i} + xz\vec{j} + xy\vec{k}$ over the path $\vec{r}(t) = (t + \sin(10\pi t)e^t, t^2 - \cos(2\pi t), 2^t)$ as $t$ varies from 0 to 2.

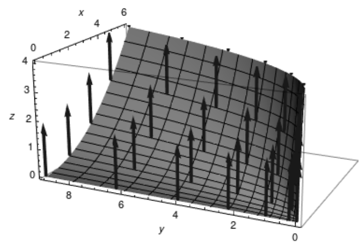


**Problem 6** (M6). (a) Let $\vec{F}(x,y,z) = \sqrt{x^5 + x}\vec{i} + (x^2yz - z)\vec{j} + (x\sqrt{z^3 + y} + y)\vec{k}$. Compute the flux of the vector field $\nabla \times F$ through a net whose rim is the unit circle $y^2 + z^2 = 1$ in the $x = 0$ plane, oriented in the $\vec{i}$ direction.

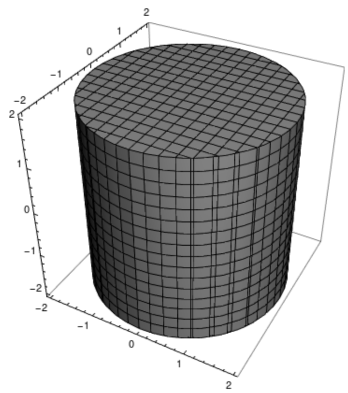



(b) Find the flux of the vector field $\vec{F}(x,y,z) = (x, xy, z)$ through the surface parametrized by $\vec{r}(s,t) = (st, s^2, t^2)$ oriented upwards, for $0 \le s \le 3, 0 \le t \le 2$.

*Note: the arrows in the diagram are the orientation of the surface, not a representation of $F$.*




(c) Compute $\int_S \vec{F} \cdot d\vec{A}$, where $\vec{F}(x, y, z) = xy^2\vec{i} + x^2y\vec{j} + (x^2y^2 + z)\vec{k}$ and $S$ is the surface (including both ends!) of a closed cylinder with radius 2 centered on the $z$-axis, from $z = -2$ to $z = 2$.




**Problem 7** (S5)**.** Let

$$\vec{F}(x, y, z) = (0, x, y) \qquad \vec{G}(x, y, z) = (2x, z, y) \qquad \vec{H}(x, y, z) = (3y, 2x, z).$$

(a) For each field, either find a scalar potential function or prove that none exists.

(b) For each field, either find a vector potential function or prove that none exists.