Math 2233 Practice Midterm 1 Solutions
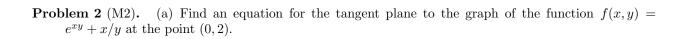
Instructor: Jay Daigle

(a) These are the instructions you will see on the real test, next week. I include them here so you know what to expect.

(b) You will have **90** minutes for this test.

(c) You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.

(d) You may use a calculator, but don't use a graphing calculator or anything else that can do symbolic computations. Using a calculator for basic arithmetic is fine, but will probably hurt you.

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Problem 1 (M1). (a) Find the area of the triangle with vertices $(4, 1, 1), (3, 2, 2), (2, 3, 4)$.

(b) Find the cosine of the angle between the vectors $\vec{v} = 3\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{u} = \vec{i} - 2\vec{j} + \vec{k}$.
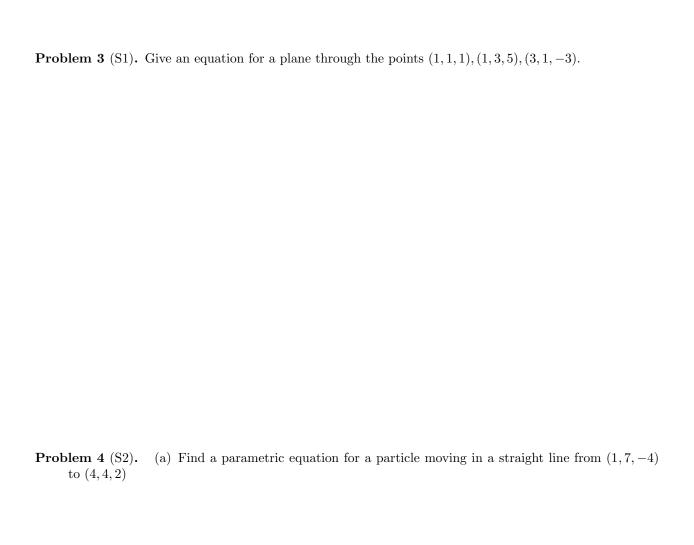
(c) Let $\vec{v} = 3\vec{i} + \vec{j} - \vec{k}$ and $\vec{u} = -2\vec{i} - \vec{j} + 2\vec{k}$. Compute the orthogonal decomposition of $\vec{v}$ with respect to $\vec{u}$. That is, write $\vec{v} = \vec{v}_{\text{parallel}} + \vec{v}_{\perp}$.

Problem 2 (M2).   (a) Find an equation for the tangent plane to the graph of the function $f(x, y) = e^{xy} + x/y$ at the point $(0, 2)$.

(b) Let $g(x, y, z) = x^2y + y^2z$. Use a linear approximation at the point $(1, 2, 3)$ to estimate $g(.9, 2.1, 3.2)$.

(c) Let $h(x, y) = 2xy - x^2y - 2$, and $\vec{u} = \frac{-3}{5}\vec{i} + \frac{4}{5}\vec{j}$. Compute $h_{\vec{u}}(2, 1)$.

(d) Compute $\nabla\left(x^2z + \sqrt{xy}\right)$. At the point $(1, 2, 1)$, which direction should we move to increase the value of this function as quickly as possible?

Problem 3 (S1). Give an equation for a plane through the points $(1, 1, 1), (1, 3, 5), (3, 1, -3)$.

Problem 4 (S2).   (a) Find a parametric equation for a particle moving in a straight line from $(1, 7, -4)$ to $(4, 4, 2)$

(b) Suppose another particle follows the path $\vec{r}_2(t) = (4t, t + 3, t^2 + t)$. Does this particle's path intersect the path of the particle from part (a)?

Problem 5 (S3). Let $f(x, y) = 2xy - x^2y - 2$

(a) Sketch and clearly label cross-sections of f for $x = -1, 0, 1$ and $y = -2, 0, 2$.

(b) Sketch and clearly label contours of f for $c = -4, -2, 0$.