Math 2233 Summer 2025
Multivariable Calculus
Mastery Quiz 1
Due Wednesday, July 2

This week's mastery quiz has two topics. Everyone should submit both M1 and S1.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1:
- Secondary Topic 1:

Name:

Name:

M1: Vectors

(a) Let $\vec{v} = 3\vec{j} - 2\vec{k}$. If we start at the point $(2,1,1)$ and then follow the vector $-2\vec{v}$, where do we end?

Solution: We wind up at the point $(2,1,1) - 6\vec{j} + 4\vec{k} = (2,-5,5)$.

(b) Find the area of the parallelogram with vertices $(0,0,0), (2,2,2), (1,5,5), (3,7,7)$.

Solution: This parallelogram is spanned by the vectors $2\vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{i} + 5\vec{j} + 5\vec{k}$, so we compute

$$\begin{aligned}\left(2\vec{i} + 2\vec{j} + 2\vec{k}\right) \times \left(1\vec{i} + 5\vec{j} + 5\vec{k}\right) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 2 \\ 1 & 5 & 5 \end{vmatrix} \\ &= 10\vec{i} + 2\vec{j} + 10\vec{k} - 2\vec{k} - 10\vec{i} - 10\vec{j} \\ &= -8\vec{j} + 8\vec{k}.\end{aligned}$$

Thus the area of the parallelogram is $\| - 8\vec{j} + 8\vec{k}\| = \sqrt{64+64} = \sqrt{128} = 8\sqrt{2}$.

(c) Find the orthogonal decomposition of $\vec{v} = 2\vec{i} - 3\vec{j} + 5\vec{k}$ with repsect to $\vec{u} = \vec{i} + 3\vec{j} - 2\vec{k}$.

Solution: First we compute the projection

$$\begin{aligned}\operatorname{Proj}_{\vec{u}} \vec{v} &= \frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u} \\ &= \frac{-17}{14}(\vec{i} + 3\vec{j} - 2\vec{k}) \\ &= \frac{-17}{14}\vec{i} - \frac{51}{14}\vec{j} + \frac{34}{14}\vec{k}\end{aligned}$$

Now we still need the perpendicular component, but this is a straightforward subtraction:

$$\begin{aligned}\vec{v}_\perp &= \vec{v} - \operatorname{Proj}_{\vec{u}} \vec{v} \\ &= 2\vec{i} - 3\vec{j} + 5\vec{k} - \left(\frac{-17}{14}\vec{i} - \frac{51}{14}\vec{j} + \frac{34}{14}\vec{k}\right) \\ &= \frac{45}{14}\vec{i} + \frac{9}{14}\vec{j} + \frac{36}{14}\vec{k}.\end{aligned}$$

S1: Lines and Planes

(a) Find an equation for the plane that passes through the points $(0,-1,0)$, $(7,3,-1)$, and $(4,2,-2)$

Solution: We can find two vectors in the plane, e.g. $\vec{u} = 7\vec{i} + 4\ vecj - \vec{k}$ and $\vec{v} = 4\vec{i} + 3\vec{j} - 2\vec{k}$. Then teh cross product is

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 4 & -1 \\ 4 & 3 & -2 \end{vmatrix} = -8\vec{i} - 4\vec{j} + 21\vec{k} - \left(16\vec{k} - 3\vec{i} - 14\vec{j}\right)$$
$$= -5\vec{i} + 10\vec{j} + 5\vec{k}$$

and thus we get the equation

$$-5(x-0) + 10(y+1) + 5(z-0) = 0.$$

(b) Find a vector perpendicular to the plane given by the equation

$$-2(x-3) + 6(y+1) + 4(z+8) = 0$$

Solution: A normal vector is $-2\vec{i} + 6\vec{j} + 4\vec{k}$.

(c) Find an equation for the plane perpendicular to $\vec{n} = \vec{3}i - \vec{2}j + \vec{4}k$ that passes through the point $(2, 1, -1)$.

Solution:

$$3(x-2) - 2(y-1) + 4(z+1) = 0.$$