

Math 2233 Summer 2025
Multivariable Calculus
Mastery Quiz 10
Due Monday, August 4

This week's mastery quiz has **two** topics. Everyone should submit both.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 5: Line Integrals
- Secondary Topic 5: Vector Fields

Name:

M5: Line Integrals

- (a) Compute the arc length of the curve $\vec{r}(t) = (t^2, 3, \frac{1}{3}t^3)$ between the points $(0, 3, 0)$ and $(1, 3, 1/3)$.
- (b) Let $f(x, y, z) = xy + zx^2$. Compute $\int_C \nabla f \, ds$ where C is parametrized by the curve $\vec{r}(t) = (2t - 1, t^4 + t, \sin(\pi t))$ for $t \in [0, 2]$.
- (c) Let C be the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.
Compute the line integral of the vector field $\vec{F}(x, y) = (xy, -x^2)$.

S5: Vector Fields

- (a) Find a flow line for the vector field $\vec{F}(x, y) = 3\vec{i} + -6x\vec{j}$ that goes through the point $(1, 2)$ at time $t = 0$.
- (b) If $H(x, y, z) = xyz\vec{i} + \sin(xy)\vec{j} - \cos(yz)\vec{k}$, compute $\nabla \times H(x, y, z)$.
- (c) Find a potential function for $\vec{F}(x, y) = (x + y)\vec{i} + (x - y)\vec{j}$ or prove none exists.
- (d) Find a potential function for $\vec{F}(x, y, z) = y\vec{i} + x\vec{j} + xz\vec{k}$ or prove none exists.