Math 2233 Summer 2025
Multivariable Calculus
Mastery Quiz 11
Due Wednesday, August 6

This week's mastery quiz has three topics. Everyone should submit M6. If you have a 4/4 on M5, or a 2/2 on S5, you don't need to submit them again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 5: Line Integrals
- Major Topic 6: Surface Integrals
- Secondary Topic 5: Vector Fields

Name:

Name:

M5: Line Integrals

(a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y) = x^2y^2\vec{i} + 2xy\vec{j}$ and C is the boundary of the square (clockwise) through points $(0,0), (0,3), (3,3), (3,0)$.

(b) Evaluate $\int_C \vec{G} \cdot d\vec{r}$ where $\vec{G}(x,y,z) = z\vec{i} - y\vec{j} + 2x\vec{k}$ and C is the straight line path from $(0,0,0)$ to $(1,2,3)$.

Name:

(c) Let $f(x, y, z) = 3x^2y + \frac{z}{x}$. Compute $\int_C \nabla f \, ds$ where C is parametrized by the curve $\vec{r}(t) = (t, \frac{t^2}{t+4}, t)$ for $t \in [1, 4]$.

Name:

M6: Surface Integrals

(a) Find the surface area of the part of the paraboloid $z = 16 - x^2 - y^2$ which lies above the xy-plane.

(b) Compute the flux of the vector field $\vec{F}(x, y, z) = y\vec{i} + z\vec{k}$ upwards through portion of the cone $z = 1 - \sqrt{x^2 + y^2}$ above the plane $z = 0$.

(c) Compute $\iint_S \nabla \times \vec{F} \cdot d\vec{S}$ where $\vec{F}(x, y, z) = (z - y, x, -x)$ and S is the hemisphere $x^2 + y^2 + z^2 = 4, z \geq 0$, oriented inwards towards the center of the hemisphere.

Name:

S5: Vector Fields

(a) Which of the following is a flow line for $\vec{F}(x, y, z) = 2x\vec{i} + z\vec{j} - z^2\vec{k}$?

(i) $\vec{r}(t) = \left(e^{2t}, \ln|t|, \frac{1}{t}\right)$

(ii) $\vec{s}(t) = \left(e^t, \frac{1}{t}, t\right)$

(b) Find a potential function for $\vec{G}(x, y) = xy\vec{i} + (x + y)\vec{j})$, or prove none exists.

(c) Find a potential field for $\vec{F}(x, y, z) = (y + z)\vec{i} + (x + z^2)\vec{j} + (x + 2yz)\vec{k}$ or prove none exists.