Math 2233 Summer 2025
Multivariable Calculus
Optional Mastery Quiz 12
Due Friday, August 8
Online by midnight

This week's mastery quiz has two topics. If you have a 4/4 on M5, you don't need to submit it again. If you have **2/2 on M6** you don't need to submit it again; because we have so little time to quiz it I'm grading it on only the best score.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

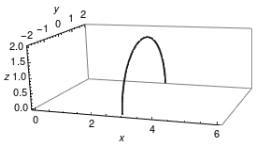
- Major Topic 5: Line Integrals
- Major Topic 6: Surface Integrals

Name:

Name:

M5: Line Integrals

(a) Set up, but **do not evaluate**, an integral to compute: the mass of a wire following a semi-circular path of radius 2 contained in the $x = 3$ plane, which goes from $(3, 2, 0)$ through $(3, 0, -2)$ to $(3, -2, 0)$, with density given by $\delta(x, y, z) = x^2 + y^2 + \sqrt{z^2 + 1}$?



(b) Compute $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z) = (x^2 - y, y^2 + x, 1)$ and C is the intersection of $x^2 + y^2 = 1$ and $z = y^2$, **oriented counterclockwise when** viewed from the point $(0, 0, 100)$.

(c) Let $f(x, y, z) = x^3y - xz^2$. Compute $\int_C \nabla f \, ds$ where C is parametrized by the curve $\vec{r}(t) = (t + 1, t^2 - 2, \sin(\pi t))$ for $t \in [0, 2]$.

M6: Surface Integrals

(a) Let V be the volume between the spherical shells of radius 1 and 2 centered at the origin. Compute the flux of the vector field $\vec{F}(x, y, z) = x\vec{i} + y\vec{j}$ out of the volume V.

(b) The moment of inertia of a surface about the z axis is given by the formula $I = \iint_S x^2 + y^2 \, dS$. Find the moment of inertia of the surface $z = xy$ lying inside the cylinder $x^2 + y^2 = 3$.

(c) Let S be the boundary of the cube with opposite corners $(0, 0, 4)$ and $(-4, -4, 0)$. Compute the flux of the vector field $\vec{F}(x, y, z) = x^2y\vec{i} - xyz\vec{j} + x^3y^2\vec{k}$ outwards through the surface S.