Math 2233 Summer 2025
Multivariable Calculus
Mastery Quiz 4
Due Monday, July 14

This week's mastery quiz has three topics. Everyone should submit M3, which you're seeing for the second time, and M4, which is new. If you have a 4/4 on M2 (meaning you have gotten 2/2 twice), you don't need to do it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Partial Derivatives
- Major Topic 3: Optimization
- Major Topic 4: Integration

Name:

M2: Partial Derivatives

(a) Give an equation for the plane tangent to $f(x, y) = 3 + x^2y + \ln(x + xy)$ at the point $(1, 0)$.

(b) Find all three second partial derivatives of $g(x, y) = \sqrt{x^2 + y}$.

(c) Let $f(x, y, z) = \frac{e^x\sqrt{y}}{z}$. Find the directional derivative in the direction $\vec{i} + 3\vec{j} - 2\vec{k}$ at the point $(0, 1, 1)$.

M3: Optimization

(a) Find and classify the critical points of $f(x, y) = x^2 + xy^2 + y^2$.

(b) Find the maximum and minimum values of $f(x, y) = xy$ subject to the constraint that $x^2 + 4y^2 \leq 1$.

Name:

M4: Integration

(a) Find the volume of the solid bounded above by $f(x, y) = e^{-x^2}$, below by the plane $z = 0$, and over the triangle formed by $x = 1, y = 0, y = x$.

(b) Sketch the region of integration and compute $\int_{-1}^{0} \int_{-\sqrt{y+1}}^{\sqrt{y+1}} y^2 \, dx \, dy$. (Do not use a calculator!)

(c) Integrate the function $f(x, y, z) = xyz$ over the region bounded by $x = 0, x = 4, y = 0, z = 0$, and $y = 4 - z^2$.