Math 2233 Summer 2025
Multivariable Calculus
Mastery Quiz 7
Due Wednesday, July 23

This week's mastery quiz has two topics. Everyone should submit M4, which you're seeing for the second time. If you have a 4/4 on M3 (meaning you have gotten 2/2 twice), you don't need to do it again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 3: Optimization
- Major Topic 4: Integration

Name:

M3: Optimization

(a) Find and classify the critical points of $f(x, y) = x^2 + xy^2 + y^2$.

(b) Use the method of Lagrange multipliers to find the point on the circle $x^2 + y^2 = 40$ closest to the point $(1, 3)$.

(c) Find (but don't classify) the critical points of $g(x, y, z) = x^3 + y^3 - 3x^2 - y^2 - z^2 + 2z - 1$.

M4: Integration

(a) We want to find the volume of the region enclosed by the portion of the cylinder $x^2 + y^2 = 9$ with $y \leq 0, z \geq 0$, and the sphere $x^2 + y^2 + z^2 = 25$. Set up three different iterated integrals to compute this, in cartesian, cylindrical, and spherical coordinates. Choose one of the integrals you set up and evaluate it.

(b) Sketch the region of integration and compute $\iint_R y\sqrt{x^2 + y^2}\, dA$ where R is the region given by $x^2 + y^2 \leq 4$ and $0 \leq y \leq x$.

(c) Let R be the parallelogram with vertices $(0, 0), (1, 2), (3, 3), (4, 5)$. Find a transformation that translates to the square with vertices $(0, 0), (0, 1), (1, 0), (1, 1)$. Use this transformation to compute $\iint_R xy\, dA$.