Math 2233 Summer 2025
Multivariable Calculus
Mastery Quiz 8
Due Monday, July 28

Sorry, I know I posted this late. I'll still accept it on Tuesday, but if you can get it in on Monday we'll both find the rest of the week easier.

This week's mastery quiz has three topics. Everyone should submit S4, unless you nailed it on the midterm. (Check Blackboard!) If you have a 4/4 on M3 or M4, (meaning you have gotten 2/2 twice), you don't need to do them again.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 3: Optimization
- Major Topic 4: Integration
- Secondary Topic 4: Integral Applications

Name:

Name:

M3: Optimization

(a) Find and classify the critical points of $f(x, y) = 2x^3 - 6xy + y^2$.

(b) Find the maximum and minimum values of $g(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.

Name:

(c) Find (but don't classify) the critical points of $g(x, y, z) = x^2 + y^2 + 3z^2 - 2x - 8y - z^3 + 5$.

Name:

M4: Integration

(a) We want to integrate the function $f(x, y, z) = (x^2+y^2+z^2)^{3/2}$, over the region enclosed by the cone $z = \sqrt{3x^2 + 3y^2}$ and the sphere $x^2 + y^2 + z^2 = 16$. Set up three different iterated integrals to compute this, in cartesian, cylindrical, and spherical coordinates.

Choose one of the integrals you set up and evaluate it.

(b) Use the change of variables $s = y, t = y - x^2$ to evaluate $\iint_R x\, dA$ over the region in the first quadrant bounded by $y = 0, y = 36, y = x^2, y = x^2 - 1$.

(c) Sketch the region of integration and compute $\iint_R xy^2\, dx\, dy$, where R is the region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$. (Do not use a calculator!)

Name:

S4: Integral Applications

(a) Find the mass of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$, and $x + 2y + 3z = 6$ if the density is given by $\delta(x, y, z) = z$.

(b) Let R be a trapezoidal lamina bounded by the lines $y = -x/4+5/2, y = 0, y = 2, x = 0$, with density $\rho(x, y) = y^2$.

(i) Sketch a picture of R.

(ii) Find the mass of R.

(iii) Find the center of mass of R.

(iv) Find the moments of inertia of R.