Syllabus and Mathematical Reasoning

Jay Daigle

jaydaigle@gwu.edu

https://jaydaigle.net/politics

The George Washington University

What are we doing here?

- Math isn't just numbers and equations
  - Identify our assumptions
  - Describe them explicitly
  - Understand implications
- Mathematical ideas can explain political systems
  - How do we decide who wins elections?
  - How do we allocate political power?
  - How do we negotiate successfully?
- Approach questions like a mathematician
  - Articulate our thoughts clearly
  - Understand our own priorities
  - Converse with and persuade effectively

Boring Logistics

Textbook

The Mathematics of Politics, Second Edition

by E. Arthur Robinson and Daniel H. Ullman

`https://wrlc-gwu.primo.exlibrisgroup.com/permalink/01WRLC_GWA/1j51gk4/alma99185918062204107`

Course Web Page

`https://jaydaigle.net/politics/`

Linked from Blackboard

Contacting me

Me

Jay Daigle

jaydaigle@gwu.edu

Office Hours

Office: Phillips 720E

Office hours:

Tuesday Noon–2 PM

Unit 1: Voting

Aug 24	Mathematical Reasoning	Aug 26	Voting Systems
Aug 31	Two-Candidate Elections	Sept 2	Criteria for Voting Systems, **Quiz**
~~Sept 7~~	*No Class for Labor Day*	Sept 9	Multi-Candidate Criteria
Sept 14	Theorems on Voting System Criteria	Sept 16	Evaluating Voting Systems I
Sept 21	Evaluating Voting Systems II, **Quiz**	Sept 23	Arrow's Impossibility Theorem
Sept 28	**Midterm 1**		

Unit 2: Apportionment

		Sept 30	The Problem of Apportionment
Oct 5	Hamilton's Method	Oct 7	Jefferson's Method
~~Oct 12~~	*No Class for Fall Break*	Oct 14	Divisor Methods, **Quiz**
Oct 19	More on Divisor Methods	Oct 21	Evaluating Apportionment Methods
Oct 26	Criteria and Impossibility, **Quiz**	Oct 28	The Best Methods
Nov 2	**Midterm 2**		

Unit 3: Conflict and Game Theory

		Nov 4	Zero-Sum Games
Nov 9	Strategies and Saddle Points	Nov 11	Probability and Randomness
Nov 16	Mixed Strategies, **Quiz**	Nov 18	Nash Equilibria
~~Nov 23~~	*No Class for Thanskgiving Break*	~~Nov 25~~	*No Class for Thanskgiving Break*
Nov 30	Solving 2×2 games	Dec 2	Conflict and Cooperation
Dec 7	Important Games, **Quiz**	Dec 9	Class Choice

Course Structure

Grade Distribution

- Homework: 20%
  (drop two lowest)
- Tests: 30% (15% each)
- Quizzes: 25%
  (drop lowest, 5% each)
- Final Exam: 25%

Homework

- Written or typed
- Due every Tuesday on Blackboard
- Must be your own work
- Two lowest scores are dropped
- For your benefit, so you can practice

Quizzes and Tests

Quiz schedule

- Six quizzes, two on each unit
- Timed, in-class
- Usually about 20 minutes
- Check your understanding of the material as we go

Test Schedule

- Test 1 on Voting: Monday, September 28
- Test 2 on Apportionment: Monday, November 2
- Final exam: As scheduled by registrar

Do not book travel before finals are over!

Mathematical reasoning

1. Definitions
2. Proofs

Definitions

- Precise, not fuzzy
- Taken literally
- Not normal English meanings!
- Similar to legal reasoning

Is a bee a fish?

Each of these statutes provides that covered species include "native species or subspecies of a bird, mammal, fish, amphibian, reptile, or plant[.]" This portion of the code, however, does not elaborate on what qualifies as a bird, mammal, fish, and so forth. Based only on the qualified species listed above, bees and other land-dwelling invertebrates would not receive protection under the law. The court looked elsewhere in the Fish and Game Code for definitions to help clarify whether bees may qualify for protection under CESA.

Is a bee a fish?

Importantly, the section 45 of the code defines "fish" as "a wild fish, mollusk, crustacean, invertebrate, amphibian, or part, spawn, or ovum of any of those animals." (Emphasis added). According to the court, the term "invertebrate" under the definition of fish includes both aquatic and terrestrial invertebrates, such as bees.

Definition of a function

Definition

A function is a rule that assigns exactly one output to every valid input. We call the set of valid inputs the domain of the function, and the set of possible outputs the codomain or sometimes the range. A function must be deterministic, in that given the same input it will always yield the same output.

Definition (Unnecessarily technical definition)

Let A, B be sets. We define a function $f : A \to B$ to be a set of ordered pairs $\{(a, b) : a \in A, b \in B\}$ such that for each a in A there is exactly one pair whose first element is a. We call A the domain and B the codomain of f.

Proofs

Types of theorems

- "Theorem" for a major, important result
- "Proposition" for a less important result that we still care about
- "Lemma" for annoying technical results we mainly want in order to prove something else we actually care about
- "Corollary" for something that follows immediately from something we've already proven.

Theorems are universal

- Need universal arguments to prove something is always true
- Need one counterexample to prove something is not always true
- (Need universal arguments to prove something is never true)

Example

- All swans are white
- No swans are red
- No black swan is white

Hypotheses: When Things Break

A theorem tells you when something can break

Hypotheses are conditions that force a result to be true.

Example

If we decrease revenue and increase spending, the deficit will increase.

Example

A voting method that is anonymous and neutral cannot be decisive.

Theorems tell us about tradeoffs we can't avoid.

Discussion Question

Alex, Bailey, and Casey are running for office.

33 voters like Alex best	18 prefer Bailey to Casey
	15 prefer Casey to Bailey
32 voters like Bailey best	24 prefer Alex to Casey
	8 prefer Casey to Alex
34 voters like Casey best	16 prefer Alex to Bailey
	18 prefer Bailey to Alex

Question

Who should win?

Another view

18	15	24	8	16	18
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

Questions to think about

- Who has the most first-place votes? C has 34
- Who has the least first-place votes? B has 32
- Who has the most *last*-place votes? C has 42
- Who has the least last-place votes? A has 26

Another view

18	15	24	8	16	18
A	A	B	B	C	C
B	C	A	C	A	B
C	B	C	A	B	A

More Questions

- Who wins between A and B? B wins 50 to 49
- Who wins between A and C? A wins 57 to 42
- Who wins between B and C? B wins 50 to 49