Voting Systems

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Terminology

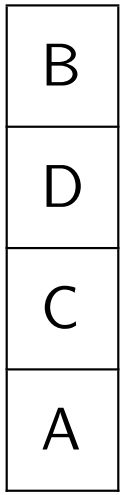
Consider an election with multiple candidates

Definition

The set of candidates is the slate. A, B, C, ...

The set of voters is the electorate.

Each voter submits a preference ballot, listing candidates in descending order of preference.



Basic Assumptions

Logistics: not our problem

We assume the votes are already collected and counted

- Who gets to vote?
- How do they submit their votes?
- What if voters make mistakes?
- What if we count the votes wrong?

Important questions! But we'll ignore them.

Basic Assumptions

Rational voters

We assume voter preferences rational or transitive

If $B > D$ and $D > C$ then $B > C$.

True for numbers. Is it true for people?

Voter profiles

A	B	A	C	A	B	C
B	C	B	A	B	C	A
C	A	C	B	C	A	B

Tabulated Profiles

3	2	2
A	B	C
B	C	A
C	A	B

Social Choice Functions

Definition

A social choice function for a slate of candidates takes in a voter profile, and outputs a non-empty subset of the slate.

We think of this subset as the list of "winners" of the election. We must have at least one winner, but we can have multiple winners (which you might interpret as a tie).

- Lots of possible functions!
- With three candidates and four voters, number of possible functions is a thousand digit number.
- But most aren't very interesting.

Plurality Voting

Definition

A candidate who gets more votes than any other candidate is said to have a plurality of the votes.

In the plurality method the winner is the candidate who is ranked first by the largest number of voters.

If there is a tie for the most first-choice votes, all the candidates who tie for the most first-choice votes are winners.

- The good: simple and familiar
- The bad: doesn't use much information!

Plurality Voting Example

3	2	2
A	B	C
B	C	A
C	A	B

Who wins?

A has three first-place votes, while B and C each have two. So A wins.

Plurality Voting Problems

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

Who wins?

A has five first-place votes, while B, C, and D each have four and E has three. So A wins.

But fifteen people hate A. Is this a good idea?

Hare's Method

Definition (Hare's Method)

- Unless all candidates have the same number of first-place votes, identify the candidate (or candidates) who have the fewest first-place votes, and eliminate them from consideration.
- Each voter moves their lower-ranked candidates up one place.
- Repeat, either until only one candidate remains, or until all remaining candidates have the same number of first-place votes. The remaining candidates are the winners.

Hare's Method

Hare's Method

- Used in Australia, Papua New Guinea, Alaska, Maine, NYC
- Rewards strong base of support
- Simulates multi-round election with runoffs
- Referred to as *Instant Runoff Voting*, *Single Transferable Vote*
- Sometimes just *ranked choice voting*

  But lots of voting methods use ranked choice!

Hare's Method in Maine

- 2026 Maine Democratic Gubernatorial Primary

Candidate	Round 1	Round 2	Round 3	Round 4
Pingree	50,552 (23%)	55,360 (26%)	75,671 (36%)	111,750 (56%)
Shah	58,606 (27%)	62,860 (30%)	72,681 (35%)	86,950 (44%)
Jackson	45,959 (21%)	47,597 (22%)	60,010 (29%)	Eliminated
Bellows	44,770 (21%)	47,049 (22%)	Eliminated	—
King III	17,860 (8%)	Eliminated	—	—
Exhausted	—	4,881 (2%)	9,385 (4%)	19,047 (9%)
Continuing	217,747	212,866	208,362	198,700

- Table from Andrew Gelman's blog

Hare's Method Example

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

$\rightarrow^{E}$

5	4	4	4	3
A	B	C	D	D
B	C	B	B	B
C	D	D	C	C
D	A	A	A	A

$\rightarrow^{B,C}$

5	4	4	4	3
A	D	D	D	D
D	A	A	A	A

$=$

5	15
A	D
D	A

So D wins.

Who would win in a race between just B and D?

Hare's Method Example 2

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

$\rightarrow^{E}$

5	4	4	4	3
B	C	A	D	A
C	A	B	A	B
D	B	D	B	D
A	D	C	C	C

$\rightarrow^{C,D}$

5	4	4	4	3
B	A	A	A	A
A	B	B	B	B

$=$

5	15
B	A
A	B

So A wins.

Coombs's Method

Definition (Coombs's Method)

- Unless all candidates have the same number of last-place votes, identify the candidate (or candidates) with the most last-place votes, and eliminate them.
- As in Hare's method, each voter moves their lower-ranked candidates up one place.
- Repeat, either until only one candidate remains, or until all remaining candidates have the same number of last-place votes. The candidate or candidates who remain at the end are the winners.

Rewards lack of strong opposition

Coombs's Method Example

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

$\rightarrow^{A}$

5	4	4	4	3
B	B	C	D	E
C	C	B	B	D
E	E	D	E	B
D	D	E	C	C

$=$

9	4	4	3
B	C	D	E
C	B	B	D
E	D	E	B
D	E	C	C

$\rightarrow^{D}$

9	4	4	3
B	C	B	E
C	B	E	B
E	E	C	C

$\rightarrow^{E}$

9	4	4	3
B	C	B	B
C	B	C	C

$=$

16	4
B	C
C	B

So B wins.

Coombs's Method Example 2

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

$\rightarrow^{C}$

5	4	4	4	3
B	A	A	D	E
E	B	B	A	A
D	E	E	B	B
A	D	D	E	D

$=$

5	8	4	3
B	A	D	E
E	B	A	A
D	E	B	B
A	D	E	D

$\rightarrow^{D}$

5	8	4	3
B	A	A	E
E	B	B	A
A	E	E	B

$=$

5	12	3
B	A	E
E	B	A
A	E	B

$\rightarrow^{E}$

5	12	3
B	A	A
A	B	B

$=$

5	15
B	A
A	B

so A wins.

Borda Count

Definition (Borda Count)

- If there are n candidates, give each candidate $n-1$ points for each voter who ranks them first; $n-2$ points for each voter who ranks them second; $n-3$ points for each candidate who ranks them third; and so on, until they get 1 point for each voter who ranks them second-to-last (and 0 points for each voter who ranks them last.)
- Add up all the points; the candidate who gets the most points wins.
- If more than one candidate ties for the most points, all of them win.

Rewards broad moderate support

Borda Count Example

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

	5	4	4	4	3
4	A	B	C	D	E
3	B	C	B	B	D
2	C	E	D	E	B

- A gets 20 points
- B gets 61 points
- C gets 45 points
- D gets 37 points
- E gets 37 points.

B wins by the Borda count method.

Borda Count Example 2

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

- A gets 49 points
- B gets 54 points
- C gets 31 points
- D gets 28 points
- E gets 38 points.

B wins by the Borda count method.

Questions

- Who wins by plurality?
- Who wins in a race between A and B?

Copeland's Method

Definition (Copeland's Method)

- Each candidate earns one point for every candidate they beat in a head-to-head matchup (by simple majority). A candidate earns half a point for every candidate they tie.
- The candidate with the most points at the end wins. If there's a tie for the most points, all those candidates are winners.

Rewards strong head-to-head matchups

Rewards candidates who are preferred to many other individual candidates

Copeland's Method Example

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

	A	B	C	D	E	Total
A	-	0	0	0	0	0
B	1	-	1	1	1	4
C	1	0	-	1	1	3
D	1	0	0	-	0	1
E	1	0	0	1	-	2

	AvB	AvC	AvD	AvE	BvC	BvD	BvE	CvD	CvE	DvE
1st	5	5	5	5	16	13	17	13	13	8
2nd	15	15	15	15	4	7	3	7	7	12
Winner	B	C	D	E	B	B	B	C	C	E

B wins with four total points.

Copeland's Method Example 2

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

	A	B	C	D	E	Total
A	-	1	1	1	1	4
B	0	-	1	1	1	3
C	0	0	-	0	0	0
D	0	0	1	-	0	1
E	0	0	1	1	-	2

A wins with four points.

Slate 1 scoreboard

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

Plurality	A
Hares	D
Coombs's	B
Borda Count	B
Copeland's Method	B

Slate 2 scoreboard

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

Plurality	B
Hares	A
Coombs's	A
Borda Count	B
Copeland's Method	A

Copeland's Method in Maine?

- 2026 Maine Democratic Gubernatorial Primary

Candidate A	Candidate B	% Neither	% A	% B	A vs. B Margin
Bellows	Pingree	14%	41%	44%	–3%
Bellows	Jackson	19%	48%	33%	15%
Bellows	Shah	12%	45%	43%	3%
Shah	Pingree	10%	39%	50%	–11%
Shah	Jackson	13%	50%	37%	12%
Jackson	Pingree	14%	33%	52%	–19%

- Table from Andrew Gelman's blog

Bellows	Pingree	Shah	Jackson
2	3	1	0

Which Method is Best?

- Plurality is simple
- Hare's method rewards strong support
- Coombs's method rewards lack of opposition
- Borda Count rewards broad moderate support
- Copeland's method rewards good head-to-head results

Question for you

Which is best?