Two-Candidate Elections

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Two-Candidate Elections

Definition

A social choice function for two candidates is a function whose domain is the set of all possible preferences that voters could have, and whose codomain is a set with three options: A wins; B wins; or A and B tie.

- Tempting to not allow ties
- We'll see that doesn't really work.

Voter Profiles

Voter Profile

A	B	A	B	A	A	A	A	B	B
B	A	B	A	B	B	B	B	A	A

Voter Profile

A	B	A	B	A	A	A	A	B	B

Tabulated Profile

6	4
A	B
B	A

Simple Majority

Definition (Simple Majority Method)

- The winner is the candidate who gets more than half of all the votes cast.
- If each candidate gets exactly half of the votes, then the result is a tie.

6	4
A	B

A wins.

Weighted Voting

Definition

A weighted voting method works as follows.

- There are n voters, and each voter is assigned a positive number of votes.
- We will say that voter number i has w_i votes, or a weight of w_i.
- Set $t = w_1 + w_2 + \cdots + w_n$ be the total number of votes.
- A candidate who gets more than $t/2$ votes is the winner. If no candidate gets more than $t/2$ votes, the result is a tie.

Weighted Voting Example

Unweighted Vote

A	B	A	B	A	A	A	A	B	B

A wins 6-4

Weighted Vote

5	5	5	5	1	1	1	3	3	3
A	B	A	B	A	A	A	A	B	B

- $t = 5 + 5 + 5 + 5 + 1 + 1 + 1 + 3 + 3 + 3 = 32$
- A gets $5 + 5 + 1 + 1 + 1 + 3 = 16$
- B gets $5 + 5 + 3 + 3 = 16$
- This is a tie!

The Dictatorship Method

Definition

In the dictatorship method, one of the voters is the dictator.

Whoever the dictator prefers is the winner.

Discussion Question

When does the dictatorship method make sense?

Example

Suppose we have a weighted voting method with ten voters, but the first voter gets twelve votes and each other voter gets one vote.

Then the preference of the first voter will always win the election.

Example

The company Meta has two classes of stock: Class A shares get one vote per share, and Class B shares get 10 votes per share.

Supermajority Methods

Definition

- Let p be a number such that $1/2 < p \leq 1$. The supermajority method with parameter p selects as the winner the candidate who gets a fraction p or more of all the votes.
- If there are t votes in total, a candidate must get at least $p \cdot t$ votes to win.
- If no candidate gets $p \cdot t$ votes, the result is a tie.

Example

6	4
A	B

- Take $p = 2/3$
- Need $2/3 \cdot 10 = 6\frac{2}{3}$ votes to win
- This is a tie.

Supermajority Methods

- Number of votes needed is q
- Call q the quota
- Formula for q? $q \neq pt$
- q is smallest *integer* as large as pt
- $q = \lceil pt \rceil$

Discussion Question

When does a supermajority method make sense?

Poll Question

Why don't we allow $p < 1/2$?

Status Quo Methods

Definition

- Start with some social choice function (such as the simple majority method or the supermajority method), which we will call the "base method".
- A status quo method designates one of the two candidates as the status quo, and the other as the challenger.
- If either candidate wins under the base method, then that candidate also wins under the status quo method.
- If there is a tie under the base method, then the status quo method names the status quo candidate as a winner.

Status Quo Methods

Example

6	4
A	B

- Supermajority with $p = 2/3$, B is status quo
- Supermajority calls this a tie
- B wins as the status quo candidate.

Discussion Question

When does this method get used regularly? When is it a good idea?

The Monarchy Method

Definition

In the monarchy method, one of the candidates is a monarch.

That candidate wins regardless of how anybody votes.

Remark

This is a constant function

Returns same output for any input.

Poll Question

Is monarchy the same as dictatorship?

(Remember to use *our definitions*.)

Discussion Question

When does the monarchy method make sense?

Bloc Voting

Definition

- A block voting method partitions the electorate into n blocs, so that every voter is in exactly one bloc. Then each bloc i is assigned a positive number w_i of votes.
- Each bloc conducts an election using the simple majority method, possibly with some method for resolving ties.
- Then the bloc casts all of its w_i votes in the main election for the candidate that won that simple majority election.
- The winner is the candidate who receives the most votes in the main election, with a tie if both candidates receive the same number of votes.

Bloc Voting

Example

A	B	A	B	A	B	A	B	B	A	A	A	B	A	B

- Five blocs of three voters each
- A wins blocs 1 and 4 B wins 2, 3, and 5
- B wins 3 to 2.

Tabulated profile

8	7
A	B

- Simple majority: A wins
- Can we tell the bloc voting winner?

Bloc Voting

Discussion Question

When would bloc voting make sense?

Discussion Question

Where does bloc voting get used in real life?

Bad Ideas

- Dictatorship method
- Monarchy method

Definition

In the all-ties method, the election is a tie, no matter how the electorate votes.

Definition

In the parity method, if just one candidate gets an even number of votes, then that candidate wins. If both candidates get an odd number of votes, or both candidates get an even number of votes, then the result is a tie.

Discussion Question

Can we explain *why* each method is bad?

Parity Method Example

Example

7	4
A	B

- B has an even number of votes
- A has an odd number
- B wins.

Example

What if one voter switches from B to A?

6	5
A	B

- Now A has an even number
- B has an odd number
- A wins.