Criteria for Voting Systems

Jay Daigle

jaydaigle@gwu.edu

https://jaydaigle.net/politics

The George Washington University

Evaluating Voting Methods

- We've seen a lot of voting methods
- There are lots more
- How do we decide what we want?

Discussion Question

What features should a good voting system have?

Discussion Question

What kind of election were you thinking about when you answered the last question?

Anonymity

Definition

- A method satisfies the anonymity condition (or is anonymous) if it treats all voters equally.
- An anonymous method will always give the same result if the voters exchange ballots among themselves.

Proposition

A method is anonymous if and only if its outcomes depend only on the tabulated profile.

Proof.

- "If and only if": We need to prove two separate things.

Proposition

A method is anonymous if and only if its outcomes depend only on the tabulated profile.

Proof.

- Suppose we have an anonymous method. Want to show two different profiles with the same tabulated profile will have the same result.
- Since two profiles give the same tabulated profile, same number of candidates prefer A in each profile. Can swap ballots among voters to get from first profile to second profile.
- Since the method is anonymous, swapping ballots can't change the result. So both profiles must have the same result.

Proposition

A method is anonymous if and only if its outcomes depend only on the tabulated profile.

Proof.

- Conversely, suppose we have a method that only depends on the tabulated profile.
- If voters swap ballots, that won't change the tabulated profile, so it won't change the result.
- So swapping ballots can't change the result, and the method is anonymous.

□

Neutrality

Definition

A method satisfies the neutrality criterion or is neutral if it treats both candidates equally.

Example

- Neutral: Majority, supermajority, weighted voting methods
- Also neutral: dictatorship, parity, all-ties
- Not neutral: Status Quo, Monarchy

Monotonicity

Discussion Question

- Parity method is anonymous and neutral, but obviously bad.
- What is the problem with parity? What do we want that parity doesn't give us?

Definition

- A method satisfies the monotonicity criterion or is monotone if a candidate is never hurt by getting more votes.
- That is: suppose the votes are cast and the method selects one candidate as the winner. Then suppose one or more voters change their votes from the losing candidate to the winning candidate. The candidate who was the winner before the change must remain the winner after the change.

The Problem with Parity

Proposition

The parity method is not monotone. (Or "violates monotonicity".)

Remark

- Monotone: something *always* happens
- Not monotone: something *doesn't always* happen
- (Not the same as "never happens"! Compare "it doesn't always rain" and "it never rains".)
- Just need one counterexample.

Proof.

A	B	A	B	A	A	A	A	B

A gets 6 votes and B gets 3 votes, so A wins.

The Problem with All-Ties

Discussion Question

- The all-ties method is anonymous, neutral, and monotone.
- Why is all-ties monotone?
- What's the problem with all-ties?

Definition

A method satisfies the decisiveness criterion or is decisive if it always chooses a winner, that is, never produces a tie.

Poll Question

Which of the methods we've discussed so far are decisive?

The Problem with Decisiveness

Discussion Question

A	B	A	B	B	A	B	A

- Who *should* win?
- Don't reference a specific voting method—what would we like to see happen?

Definition

A method satisfies the near decisiveness criterion or is nearly decisive if ties can only occur when both candidates receive the same number of votes.

Near Decisiveness

Definition

A method satisfies the near decisiveness criterion or is nearly decisive if ties can only occur when both candidates receive the same number of votes.

Remark

- Not exclusive.
- *Any* decisive method is nearly decisive.
- Not every nearly decisive method is decisive.

Simple Majority Method

Proposition

The simple majority method is nearly decisive.

Proof.

- Suppose the number t of voters is odd.
- No candidate can receive exactly half the votes, since $t/2$ is not an integer.
- So one must receive more than half; they win a majority and win the election.

Simple Majority Method

Proposition

The simple majority method is nearly decisive.

Proof.

- Now suppose t is even.
- If both candidates get $t/2$ votes, they receive the same number of votes, and tie.
- If not, one gets more than $t/2$ and thus has a majority and wins.
- So ties only occur when each candidate gets exactly $t/2$ votes.

□