Evaluating Voting Methods II

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Summary

Voting Methods

- Plurality
- dictatorship
- Copeland's Method
- Hare's method
- monarchy
- Borda count
- Coombs's Method
- all ties
- Antiplurality

Voting Method Criteria

- unanimous
- anonymous
- Pareto
- Condorcet
- decisive
- neutral
- independent
- anti-Condorcet
- majoritarian
- monotone

Some Theorems

Proposition

Any social choice function that satisfies anonymity and neutrality must violate decisiveness.

Proposition (Taylor)

No social choice function involving at least three candidates satisfies both independence and the Condorcet criterion.

Proposition

If a method is Condorcet then it satisfies the majority criterion.

Analyzing the Plurality Method

Proposition

The plurality method is majoritarian, monotone, and Pareto, but not Condorcet, anti-Condorcet, or independent.

The Antiplurality Method

Definition

The *antiplurality method* names as winner the candidate with the fewest last-place votes.

Example

5	4	4	4	3
A	B	C	D	E
B	C	B	B	D
C	E	D	E	B
E	D	E	C	C
D	A	A	A	A

B, C, E all win.

5	4	4	4	3
B	C	A	D	E
C	A	B	A	A
E	B	E	B	B
D	E	D	E	D
A	D	C	C	C

B and E both win.

The Antiplurality Method

Definition

The *antiplurality method* names as winner the candidate with the fewest last-place votes.

Poll Question

Which criteria does the antiplurality method satisfy?

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

The Antiplurality Method

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

Claim

The antiplurality method is monotone.

Proof.

- Raising a candidate in preference lists:
  - Can't increase their last-place votes
  - Can't decrease anyone else's last-place votes.
- If a candidate wins before getting raised, they win after.

□

The Antiplurality Method

Claim

The antiplurality method is not majoritarian, Condorcet, or anti-Condorcet.

Proof.

Consider the profile:

C	C	B	B	B
A	A	C	A	A
B	B	A	C	C

- What happens? A wins.
- B gets a majority but loses. Not majoritarian.
- B is the Condorcet winner. Not Condorcet.
- A is anti-Condorcet candidate. Not anti-Condorcet.

□

The Antiplurality Method

Claim

The antiplurality method is not Pareto.

- This is surprising!
- Just *barely* true. But true.

Proof.

Consider the profile:

A	A	A
B	B	B
C	C	C

- What happens? A and B both win.
- Every voter prefers A to B, but B wins.
- Not Pareto.

□

The Antiplurality Method

Claim

The antiplurality method is not independent.

Proof.

Consider:

C	A	A	B	B
A	B	B	A	A
B	C	C	C	C

C	A	A	B	B
A	B	B	C	C
B	C	C	A	A

- What happens?
  - A wins profile 1
  - B wins profile 2
- No voter has changed preferences between A and B.
- Not independent.

□

Hare's Method

Definition

Eliminate the candidate(s) who have the fewest first-place votes.

Repeat. The last remaining candidate(s) are the winner(s).

- Popular and widely used
- Australia, New Guinea, Alaska, Maine, NYC
- Surprisingly *bad* at our criteria.

Proposition

Hare's method is majoritarian and Pareto, but not monotone, Condorcet, anti-Condorcet, or independent.

Hare's Method

Claim

Hare's method is majoritarian.

Proof.

- A majority of first-place votes will never be the fewest
- They will never be eliminated, and everyone else will.

□

Claim

Hare's method is Pareto.

Proof.

- If everyone prefers A to B, then B has no first-place votes
- B gets eliminated in the first round, and can't win.

□

Hare's Method

Claim

Hare's method is not monotone.

Proof.

6	5	4	2
A	C	B	B
B	A	C	A
C	B	A	C

$\rightarrow$

6	5	4	2
A	C	B	A
B	A	C	B
C	B	A	C

- Profile 1: Eliminate C, then B. A wins.
- Profile 2: Eliminate B, then A. C wins.
- Getting more votes makes A lose.

Hare's Method

Claim

Hare's method is not Condorcet or anti-Condorcet.

Proof.

Consider:

2	3	2
A	B	C
C	A	A
B	C	B

- What happens?
  - Eliminate A and C; B wins.
- B is the anti-Condorcet candidate and wins
- A is the Condorcet candidate and loses.

□

Hare's Method

Claim

Hare's method is not independent.

Proof.

2	2	1
B	A	A
A	C	B
C	B	C

$\rightarrow$

2	2	1
B	C	A
A	A	B
C	B	C

- Profile 1: eliminate C, then B. A wins.
- Profile 2: eliminate A, then C. B wins.
- Relative preference of A and B has not changed

Previous Results

Proposition

The plurality method is majoritarian, monotone, and Pareto, but not Condorcet, anti-Condorcet, or independent.

Proposition

The antiplurality method is monotone, but not majoritarian, Condorcet, anti-Condorcet, Pareto, or independent.

Proposition

Hare's method is majoritarian and Pareto, but not monotone, Condorcet, anti-Condorcet, or independent.

Coombs's Method

Definition

Eliminate the candidate(s) with the most last-place votes. Repeat.

The last remaining candidate(s) are the winner(s).

Discussion Question

What criteria will Coombs's method satisfy?

Coombs's Method

Claim

Coombs's method is Pareto.

Proof.

- Assume A is ahead of B on every preference list.
- A will have no last-place votes while B is in the race.
- B will get eliminated before A does
- B can't win.

□

Coombs's Method

Claim

Coombs's method is not Condorcet or majoritarian.

Proof.

Consider:

C	C	B	B	B
A	A	C	A	A
B	B	A	C	C

- What happens?
  - Eliminate B and C; A wins.
- B is the majority candidate and loses.
- B is also the Condorcet candidate.

□

Coombs's Method

Claim

Coombs's method is not anti-Condorcet.

Wrong Argument that it is anti-Condorcet

- An anti-Condorcet candidate will lose any head-to-head
- If they make it to the last round, they'll lose that last head-to-head matchup
- An anti-Condorcet candidate can't win.

Discussion Question

What's wrong with this argument?

Coombs's Method

Claim

Coombs's method is not anti-Condorcet.

Proof.

Consider (again):

C	C	B	B	B
A	A	C	A	A
B	B	A	C	C

- What happens?
  - Eliminate B and C; A wins.
- A is anti-Condorcet but wins
- A would lose either head-to-head, but B and C are eliminated simultaneously.

□

Coombs's Method

Exercise

Coombs's method is not monotone or independent.

Proof.

- Try to do this on your own.
- Use the proofs for Hare's method for inspiration.

□

Proposition

Coombs's method is Pareto, but not majoritarian, monotone, Condorcet, anti-Condorcet, or independent.