Problem 1. (a) (2 points) Explain what it means for a voting method to be monotone. (Be precise!)

Solution: If A is winning in one profile, and then some voters change their votes to rank A higher, A will also win in the second profile.

(b) (3 points) Explain why the plurality method is monotone.

Solution: Suppose A wins an election by plurality. That means they have more first-place votes than any other candidate. If they move up in some voters' preferences, that can only increase A's number of first-place votes, and can only reduce other candidates', so A will still have the most first-place votes and will still win.

(c) (2 points) Explain what it means for a voting method to be anti-Condorcet. (Be precise!)

Solution: A method is anti-Condorcet if, whenever there's an anti-Condorcet candidate, they must be the unique winner.

Alternatively: a method is anti-Condorcet if any candidate who wins each head-to-head matchup cannot be a winner.

(d) (3 points) Show that Hare's method is not anti-Condorcet.

Solution: This requires an example. Many examples are valid but one is

2	2	3
A	B	C
B	A	A
C	C	B

Here A beats C 4 to 3, and B beats C 4 to 3, so C is anti-Condorcet. But in Hare's method, in the first round A and B tie for the fewest first-place votes and are eliminated simultaneously, so C wins despite being the anti-Condorcet candidate.

Problem 2. (a) (4 points) Suppose we have an independent voting method, which may or may not be any of the methods we've studied in class. Suppose candidate A wins and candidate B loses in the first profile below. What can you say about the result in the second profile, and why?

3	2	2
B	A	C
C	C	B
A	B	A

$\rightarrow$

3	2	2
C	A	C
B	C	B
A	B	A

Solution: Candidate B cannot win in the second profile, because the relative rankings between A and B have not changed. Since the method is independent, and A wins and B loses in the first profile, B cannot win in the second.

(b) (6 points) Suppose we have an unknown voting method, which may or may not be any of the methods we've studied in class. In the first profile below, A is the unique winner, and in the second profile, B is the unique winner. Name two criteria this method does not satisfy, and explain why.

4	2	2
A	B	C
C	C	B
B	A	A

$\rightarrow$

4	2	2
A	B	C
C	C	A
B	A	B

Solution: This method isn't monotone, because A moves up in some rankings and this causes them to lose instead of win.

This method isn't anti-Condorcet, because in the second profile B is the anti-Condorcet candidate and still wins.

(We don't know if this method is Condorcet, because there's no Condorcet candidate in either profile. We don't know if it's Pareto, because no candidate is 100% preferred to another. And we don't know if it's independent because the relative rankings of A and B *do* switch between the two profiles.)