**Problem 1.** (a) (5 points) For the following census, find the standard quota, lower quota, and Hamilton apportionment for $h = 10$.

| $k$ | $p_k$ | $q_k$ | $\lfloor q_k \rfloor$ | Hamilton $a_k$ |
|---|---|---|---|---|
| 1 | 5,700 | | | |
| 2 | 3,100 | | | |
| 3 | 1,200 | | | |

**Solution:**

| $k$ | $p_k$ | Standard Quota | Lower Quota | Hamilton Apportionment |
|---|---|---|---|---|
| 1 | 5,700 | 5.7 | 5 | 6 |
| 2 | 3,100 | 3.1 | 3 | 3 |
| 3 | 1,200 | 1.2 | 1 | 1 |

(b) Suppose we have the following census and a method that gives the following apportionments for $h = 19$. (We don't know what apportionment method is being used, and it may not be a method we've discussed in class.)

Give an apportionment for $h = 20$ that would illustrate the house paradox, and a different apportionment that would be house monotone. (You don't need to follow a specific apportionment method.)

| $k$ | $p_k$ | $h = 19$ apportionment | $h = 20$ house paradox | $h = 20$ house monotone |
|---|---|---|---|---|
| 1 | 5,000,000 | 8 | | |
| 2 | 3,000,000 | 4 | | |
| 3 | 4,500,000 | 7 | | |

**Solution:** There are lots of solutions here. One example would be this:

| $k$ | $p_k$ | $h = 19$ apportionment | $h = 20$ house paradox | $h = 20$ house monotone |
|---|---|---|---|---|
| 1 | 5,000,000 | 8 | 7 | 8 |
| 2 | 3,000,000 | 4 | 5 | 5 |
| 3 | 4,500,000 | 7 | 8 | 7 |

For the house paradox column, we need the numbers to add up to twenty, but one to be lower than the corresponding number in the $h = 19$ column.

For the house monotone column, we need the numbers to add up to twenty, and each of them at least as big as the corresponding number in the $h = 19$ column.

**Problem 2.** (a) (2 points) What does it mean if we write that $p'_k < p_k$?

**Solution:** State $k$ has a lower population in the second census than in the first census.

Alternatively: state $k$ has lost population.

(b) For the following census, find the standard quota, lower quota for each state. Compute the next Jefferson critical divisor for each state, and use them to find the Jefferson apportionment

for $h = 10$. What modified divisor did you pick? Show your modified quotas.

| | | | | | $d =$ | |
|---|---|---|---|---|---|---|
| $k$ | $p_k$ | $q_k$ | $\lfloor q_k \rfloor$ | Next CD | Modified Quota | Jefferson $a_k$ |
| 1 | 3,100 | | | | | |
| 2 | 2,500 | | | | | |
| 3 | 2,300 | | | | | |
| 4 | 2,100 | | | | | |

**Solution:** The most obvious value for $d$ is 800, which I've worked out here. But anything between 775 and 833 will work.

| | | | | | $d = 800$ | |
|---|---|---|---|---|---|---|
| $k$ | $p_k$ | $q_k$ | $\lfloor q_k \rfloor$ | Next CD | Modified Quota | Jefferson $a_k$ |
| 1 | 3,100 | 3.1 | 3 | 775 | 3.875 | 3 |
| 2 | 2,500 | 2.5 | 2 | 833 | 3.125 | 3 |
| 3 | 2,300 | 2.3 | 2 | 767 | 2.875 | 2 |
| 4 | 2,100 | 2.1 | 2 | 700 | 2.625 | 2 |