Problem 1. Consider a census in which $p_1 = 1300$, $p_2 = 3400$, and $p_3 = 5300$.

(a) (6 points) Find the Adams apportionment for $h = 10$. Explicitly give the largest and smallest divisors that will lead to the Adams apportionment. Justify your conclusion.

(b) (4 points) Find the Hill apportionment for $h = 10$. Justify your conclusion.

Solution:

(a) We need to compute critical divisors. The total population is 10,000 so the standard divisor is $s = 1000$, and the standard quotas are 1.3, 3.4, 5.3, with upper quotas 2,4,6, a total of twelve seats. We need to get rid of two.

The next standard divisors up are $1300/1 = 1300$, $3400/3 = 1133$, and $5300/5 = 1060$.

The next next standard divisors are $2400/0" = "\infty$, $3400/2 = 1700$, and $5300/4 = 1325$. So to get to ten seats, we need to be at least 1133 but below 1300.

This gives an apportionment of 2, 3, 5. We can see that either by counting critical divisors, or computing with something like $d = 1200$, which gives modified quotas of $1.08, 2.83, 4.42$.

(b) The standard divisor doesn't work here: the standard quotas round down to 1, 3, and 5, for a total of 9. We need to add one additional seat.

At this point we have two options. One is to use critical divisors. The next important critical divisors are $1300/1.414 = 919$, $3400/3.464 = 982$, and $5300/5.477 = 9.68$. Thus the "next" seat goes to state 2 when the divisor drops below 982, and the apportionment is 1, 4, 5.

Alternatively, we could guess-and-check with divisors. $d = 900$ is too small, and so is $d = 950$, but $d = 975$ gives modified quotas of $1.33, 3.49, 5.44$ which geometrically round to $1, 4, 5$.

Problem 2. Consider a census in which $p_1 = 1400$, $p_2 = 2425$, and $p_3 = 3175$, for a total population of 7000.

(a) (6 points) Find the Webster apportionment for $h = 10$. Explicitly give the largest and smallest divisors that will lead to the Adams apportionment. Justify your conclusion.

(b) (4 points) Find the Dean apportionment for $h = 10$. Justify your conclusion.

Solution:

(a) The standard divisor works here. We have $t = 7000$ so $s = 700$, and the standard quotas are $2, 3.46, 4.54$, which round to $2, 3, 5$.

To find the largest and smallest divisors that work, we will need to consider critical divisors. We compute

$1400/1.5 = 933$	$2425/2.5 = 970$	$3175/4.5 = 706$
$1400/2.5 = 560$	$2425/3.5 = 693$	$3175/5.5 = 577$

so the next critical divisor up from 700 is 706, and the next one down is 693. Any divisor between 693 and 706 will work.

(b) The standard divisor doesn't work here. For Dean's method, 2 rounds to 2, $3.46 > 3.429$ rounds up, and $4.54 > 4.444$ rounds up, so that would apportion $2 + 4 + 5 = 11$ seats.

But it's close. If we take $d = 710$, say, then we get modified quotas of $1.97 > 1.333$, $3.415 < 3.429$, and $4.47 > 4.444$, which gives an apportionment of 2,3,5.

Alternatively we can consider the Dean critical divisors. The next important Dean critical divisors are $1400/1.333 = 1050$, $2425/3.429 = 707$, and $3175/4.444 = 714$. So any divisor between 707 and 714 will work.