Math 1231 Fall 2026
Single-Variable Calculus I Section 12
Mastery Quiz Worksheet 10
Due Friday, October 30

This week's mastery quiz has three topics. It is the first time we will see topic S8 and the last quiz with topic S7. The quiz on Friday will have four questions on three topics, and should take about twenty minutes.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Friday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 3: Optimization and Extrema
- Secondary Topic 7: Curve Sketching
- Secondary Topic 8: Physical Optimization

Major Topic 3: Opitmization and Extrema

(a) The function $g(x) = (x^2 - 3x)\sqrt[3]{x-3}$ has absolute extrema either on $(-4, -1)$ or on $[1, 4]$. Pick one of those intervals, explain why g has extrema on that interval, and find the absolute extrema.

(b) Classify the critical points and relative extrema of $h(x) = \sin(x) + \cos(x)$ on $[0, 2\pi]$.

Secondary Topic 7: Curve Sketching

Let $g(x) = \dfrac{x^2 - 7}{x^2 - 4}$.

We can compute that $g'(x) = \dfrac{6x}{(x+2)^2(x-2)^2}$ and $g''(x) = \dfrac{-6(3x^2+4)}{(x^2-4)^3}$.

Sketch a graph of the function $g(x)$. Your answer should discuss the domain, asymptotes (including computing limits), roots, limits at infinity, critical points and values, intervals of increase and decrease, points of inflection, and concavity.

Secondary Topic 8: Physical Optimization

(a) Find the point on the line $y = 2x + 5$ that is closest to the origin.

(b) Suppose that a company that produces and sells x units of a product each day makes a revenue of $R(x) = 260x - 9x^2/10$ dollars per day and has costs given by $C(x) = 1000 + 100x + x^2/10$ dollars per day. What is the maximum profit that can be made (where profit is revenues minus costs)?