# Math 1231 Fall 2026
# Single-Variable Calculus I Section 12
# Mastery Quiz Worksheet 11
# Due Friday, November 6

This week's mastery quiz has three topics. It is the last time we will see topic S8 and the first time we see topic S9. The quiz on Friday will have four questions on three topics, and should take about 25 minutes.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Friday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

**Topics on This Quiz**

- Major Topic 3: Optimization and Extrema
- Secondary Topic 8: Physical Optimization
- Secondary Topic 9: Riemann Sums

# Major Topic 3: Opitmization and Extrema

(a) The function $f(x) = \dfrac{x^3 - 5x^2}{x+3}$ has absolute extrema either on the interval $[-4, -1]$ or on the interval $[-1, 4]$. Pick one of those intervals, explain why $f$ has extrema on that interval, and find the absolute extrema.

(b) Find and classify the critical points of $f(x) = \sqrt[3]{x^3 - 3x}$.

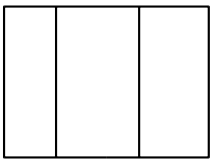
# Secondary Topic 8: Physical Optimization

(a) We wish to build a rectangular pen with two parallel internal partitions, using 1000 feet of fencing. We want to maximize the total area of the pen.

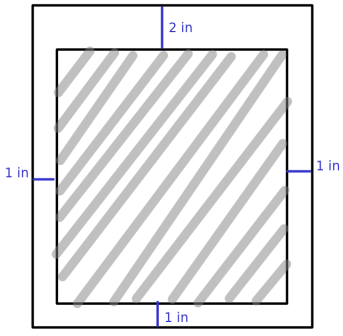
(a) What is your objective function, and why?

(b) What constraint equation(s) can you use?

(c) What dimensions maximize the total area of the pen?



(b) A poster needs to have an area of $216\text{in}^2$, with 1-inch margins on the bottom and sides and a 2-inch margin on the top. What dimension maximize the area of the *printed* region, excluding margins?



# Secondary Topic 9: Riemann Sums

Let $f(x) = x^2 - x$ be defined on the interval $[-3, 0]$.

(a) Approximate the area under the curve of the function using three rectangles and right endpoints.

(b) Approximate the area under the curve of the function using three rectangles and left endpoints.

(c) Write a formula for $R_n$, the estimate using $n$ rectangles and right endpoints, as a summation of $n$ terms.

(d) Use your answer in part (c) to find a closed-form formula for $R_n$. (This formula should not have a summation sign or be given as a sum of $n$ terms.)

(e) Use the formula in part (c) to compute the area exactly.