Math 1231: Single-Variable Calculus I
George Washington University Fall 2026
Recitation 2

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Problem 1. Let $f(x) = 5x + 2$. We want to use an $\varepsilon - \delta$ argument to compute $\lim_{x\to 2} f(x)$.

(a) If x is about 2, what should $f(x)$ be?

(b) Write down expressions using absolute value for the input and output errors.

(c) If we want $\varepsilon = 1$, what does δ need to be?

(d) Find a formula for δ in terms of ε (same form as $\delta = \varepsilon/3$ or $\delta = \varepsilon$).

(e) Try to write a full proof.

But we mostly want to practice the way we actually compute limits. Remember we have two key principles:

(a) Functions built out of algebra and trigonometry are *continuous* wherever they're defined. When a function is continuous, we can compute the limit by just plugging in.

(b) We say two functions are *almost identical* if they're the same except at a small collection of points. If two functions are almost identical, then their limits will always be the same. This allows us to replace complicated functions with simpler ones.

Problem 2 (Warmup)**.** Let $f(x) = \frac{x^2+\sin(x)+3}{x^2-x-2}$.

(a) Where is f continuous? Where is it discontinuous?

(b) What is $\lim_{x\to 0} f(x)$?

Problem 3. Let $f(x) = \frac{x-1}{x^2-1}$.

(a) What is $f(2)$? Is f continuous at 2?

(b) What is $\lim_{x\to 2} f(x)$?

(c) What is $f(1)$? Is f continuous at 1?

(d) What function can we find that's almost the same as f, but defined and continuous at 1? (Is this function the same as f?)

(e) What is $\lim_{x\to 1} f(x)$?

Problem 4. Let $g(x) = \frac{(x+1)^2-1}{x+2}$.

(a) Is g continuous where it's defined? Where is it undefined?

(b) Can you find a function that's almost identical to g but continuous everywhere?

(c) What is $\lim_{x\to -2} g(x)$?