

Math 1231: Single-Variable Calculus I
George Washington University Fall 2026
Recitation 2

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Problem 1. Let $f(x) = 5x + 2$. We want to use an $\varepsilon - \delta$ argument to compute $\lim_{x \rightarrow 2} f(x)$.

- (a) If x is about 2, what should $f(x)$ be?
- (b) Write down expressions using absolute value for the input and output errors.
- (c) If we want $\varepsilon = 1$, what does δ need to be?
- (d) Find a formula for δ in terms of ε (same form as $\delta = \varepsilon/3$ or $\delta = \varepsilon$).
- (e) Try to write a full proof.

But we mostly want to practice the way we actually compute limits. Remember we have two key principles:

- (a) Functions built out of algebra and trigonometry are *continuous* wherever they're defined. When a function is continuous, we can compute the limit by just plugging in.
- (b) We say two functions are *almost identical* if they're the same except at a small collection of points. If two functions are almost identical, then their limits will always be the same. This allows us to replace complicated functions with simpler ones.

Problem 2 (Warmup). Let $f(x) = \frac{x^2 + \sin(x) + 3}{x^2 - x - 2}$.

- (a) Where is f continuous? Where is it discontinuous?

(b) What is $\lim_{x \rightarrow 0} f(x)$?

Problem 3. Let $f(x) = \frac{x-1}{x^2-1}$.

(a) What is $f(2)$? Is f continuous at 2?

(b) What is $\lim_{x \rightarrow 2} f(x)$?

(c) What is $f(1)$? Is f continuous at 1?

(d) What function can we find that's almost the same as f , but defined and continuous at 1? (Is this function the same as f ?)

(e) What is $\lim_{x \rightarrow 1} f(x)$?

Problem 4. Let $g(x) = \frac{(x+1)^2-1}{x+2}$.

(a) Is g continuous where it's defined? Where is it undefined?

(b) Can you find a function that's almost identical to g but continuous everywhere?

(c) What is $\lim_{x \rightarrow -2} g(x)$?