Math 1231 Fall 2026
Single-Variable Calculus I Section 11
Mastery Quiz Worksheet 4
Due Friday, September 18

This week's mastery quiz has two topics. It is the first time we have seen topic S2.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, you should justify your answers and show your work. Do not just write "yes" or "no" or give a single number.

On Friday, we will end class with a short quiz, containing questions drawn from this worksheet, plus one "easy" question on topic M1. You may not use or consult any materials or notes during Wednesday's quiz; make sure you work through these problems carefully, and understand them well enough that you can replicate that work in class.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 2: Definition of Derivative

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) Compute $\lim_{x\to+\infty} \dfrac{\sqrt{3x^5+2x}}{x^{5/2}-x^{3/2}+1} =$

Solution:

$$\lim_{x\to+\infty} \frac{\sqrt{3x^5+2x}}{x^{5/2}-x^{3/2}+1} = \lim_{x\to+\infty} \frac{\sqrt{3+2/x^4}}{1-1/x+1/x^{5/2}} = \frac{\sqrt{3}}{1} = \sqrt{3}.$$

(b) Compute $\lim_{x\to-2} \dfrac{x-2}{(x+2)^2} =$

Solution:

$$\lim_{x\to-2} \frac{x-2^{\nearrow -4}}{(x+2)^2_{\searrow 0^+}} = -\infty.$$

Alternatively, we could write that the top approaches -4 and the bottom approaches 0, so the limit is $\pm\infty$. Further, we see that the top is negative and the bottom is always positive, so in fact the limit is $-\infty$.

(c) Compute $\lim_{x\to4} \dfrac{\sqrt{5-x}-1}{x-4} =$

Solution:

$$\begin{aligned}\lim_{x\to4} \frac{\sqrt{5-x}-1}{x-4} &= \lim_{x\to4} \frac{5-x-1}{(x-4)(\sqrt{5-x}+1)} \\ &= \lim_{x\to4} \frac{4-x}{(x-4)(\sqrt{5-x}+1)} \\ &= \lim_{x\to4} \frac{-1}{\sqrt{5-x}+1} = \frac{-1}{2}.\end{aligned}$$

Secondary Topic 2: Definition of Derivative

(a) **Directly from the definition of derivative**, compute $f'(-2)$ where $f(x) = \frac{1}{1-x}$.

Solution:

$$\begin{aligned}f'(-2) &= \lim_{h\to0} \frac{f(x+h)-f(-2)}{h} = \lim_{h\to0} \frac{\frac{1}{1-(-2+h)} - \frac{1}{1+2}}{h} \\ &= \lim_{h\to0} \frac{1}{h} \frac{(1+2)-(1-(-2+h))}{3(1-(-2+h))} \\ &= \lim_{h\to0} \frac{h}{h3(3-h)} = \lim_{h\to0} \frac{1}{3(3-h)} = \frac{1}{9}.\end{aligned}$$

(b) If $g(x) = \sqrt{x-5}$, find $g'(x)$, directly from the formal definition of the derivative.

Solution:

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{x+h-5}-\sqrt{x-5}}{h} \\ &= \lim_{h\to 0} \frac{x+h-5-(x-5)}{h(\sqrt{x+h-5}+\sqrt{x-5})} \\ &= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h-5}+\sqrt{x-5})} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{x+h-5}+\sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}. \end{aligned}$$